# Physics 7A-Section 2, Fall 2008. Instructor Lanzara 

FINAL EXAM
CLOSED BOOK
GOOD LUCK!
Print Name $\qquad$ Discussion Section\# or Time $\qquad$
Signature $\qquad$ Discussion Section GSI $\qquad$
Student ID\# $\qquad$

| Problem | Points | Score |
| ---: | :---: | :---: |
| 1 | 20 |  |
| 2 | 20 |  |
| 3 | 20 |  |
| 4 | 20 |  |
| 5 | 20 |  |
| 6 | 30 |  |
| TOTAL | $\mathbf{1 3 0}$ |  |

## Problem 1- [20 points]

Two blocks of mass $m$ are mounted by springs of spring constant $k$ to the center of a solid disk of mass M, radius R. These blocks are constrained to slide only radially on the disk. Initially, when the disk is at rest, the springs are at their rest length $\mathrm{r}_{0}$. Ignore friction.
a) ( 7 pts ) If we spin this system around the rotation axis, there is a maximum angular velocity, $\omega_{\text {max }}$, for which the springs can supply enough restoring force to keep the two blocks moving in circular motion. Calculate $\omega_{\text {max }}$.
b) ( 7 pts ) In reality, before reaching $\omega_{\max }$, there is an angular velocity $\omega_{\mathrm{R}}$ where the blocks would reach the outer edge of the disk. Calculate $\omega_{\mathrm{R}}$.
c) ( 6 pts ) For a given angular velocity $\omega<\omega_{\mathrm{R}}$, what is the total mechanical energy of the system?


## Problem 2- [20 points]

A thin rod of length 2 L and mass M lies flat on a frictionless plane. A small ball of mass m , traveling with velocity $\mathrm{v}_{0}$, strikes the rod at a distance $h$ from the rod's center of mass. The ball continues to move along the original line of motion after the collision (the collision is elastic) and the rod will rotate around its center of mass.
a) (12 pts) After the collision, determine the rod translational ( 4 pts ) and angular ( 4 pts ) velocity and the ball final velocity ( 4 pts ).
b) (8pts) If the collision is inelastic find the rotational angular velocity of the system after the collision.

## Top view



## Problem 3- [20 points]

A block of mass M is dropped from a height H onto a vertical spring. The spring elastic constant is k (panel a ). The block sticks to the spring and the spring compresses by L before coming momentarily to rest.
a) (4pts) How much work is done by the force of gravity while the spring is being compressed?
b) ( 4 pts ) How much work is done by the spring
c) (4pts) What was the speed of the block just before it hit the spring.
d) (8pts) Assume that, just before the block hits the spring a gun fires vertically a bullet of mass $\mathrm{m}=\mathrm{M} / 4$ that strike and remains attached to the block (panel b). If the bullet has a speed $v_{B}$ find by how much the spring is compressed this time.


## Problem 4- [20 points]

In class we saw a demonstration with a spool on a string. The spool is made of two heavy disks of radius $R_{0}$ and mass $M$, connected by a short rod of smaller radius $R_{1}$ and mass $m\left(R_{0}=5 R_{1}\right.$ and $\left.M=2 m\right)$ (panel a). The string was wrapped tightly around the rod of radius $R_{1}$. The spool rested on the table by its edges of radius $R_{0}$
a) ( 6 pts ) Describe and explain using torques in which direction the spool move and how it rotates if we pull the rope with a vertical (3pts) or horizontal (3pts) force F (panels b and c).
In class we show also that there is a specific angle $\theta_{\mathrm{c}}$ and tension $\mathrm{T}_{\mathrm{c}}$ at which the spool will spin in place (no translational velocity) at constant angular speed $\omega$. The coefficient of kinetic friction between the spool and the table $\mu_{\mathrm{k}}$ is given (panel d).
b) (10pts) Write down the three equations of motion for this special "spin-in-place" state.
c) $(2 \mathrm{pts})$ Find the angle $\theta_{\mathrm{c}}$ for this state
d) (2pts) Determine the required tension $T_{c}$.


## Problem 5- [20 points]

A laboratory rat exercises on a treadmill of mass $\mathrm{M}_{0}$ and radius $\mathrm{R}_{0}$. The axis of the treadmill is fixed and frictionless and to a good approximation all of the mass $M_{0}$ is at radius $R_{0}$. The mass of the rat is $M_{1}=1 / 2 M_{0}$ and the rat's moment of inertia about the rotation axis is $I_{1}=M_{1} R_{1}{ }^{2}$ where $R_{1}=R_{0} / \operatorname{sqrt}(2)$ is the position of the center of mass of the rat.
First the rat runs at a constant speed so that he remains stationary with respect to the laboratory and so that the treadmill rotates with constant angular velocity $\omega_{0}$. Then the rat suddenly stops running and grabs onto the treadmill with its claws.
a) (10pts) What is the angular velocity of the rat-treadmill system immediately after it grabs on?
b) ( 10 pts ) With the rat attached, the treadmill will slow as the rat is carried up and around the treadmill. What is the minimum value of $\omega_{0}$ for which the rat just makes it to the top of the treadmill?


## Problem 6- [30 points]

A bucket of section $S$ is filled up with a fluid of density $\rho_{\mathrm{L}}$ and is sitting on a scale as shown in the figure below (panel a). The top of the bucket is open to atmospheric pressure. The total reading of the scale is A.
An empty cylindrical wooden $\log$ (like a straw) is loaded with lead at one end so that it floats upright in the fluid. Both ends of the log are open. In equilibrium the length of the submerged portion is L and its distance from the bottom of the container is $h$. Neglect viscosity of the fluid and assume the fluid is uniformly distributed in the bucket.
a) ( 4 pts ) Does the weight registered by the scale go up, down or stay the same?
b) (4pts) What is the net force on the log if it is displaced upward from equilibrium by a small amount x ?
c) (6pts) Show that the $\log$ is an harmonic oscillator (3pts) and determine the period of oscillation (3pts).
Suppose now to drill a hole of section $\mathrm{S}_{1}$ into the side of the bucket at a height $d$ from the bottom of the bucket (panel b). The level of the fluid inside the bucket is $H$.
d) ( 10 pts ) Find the exit velocity of the fluid from the bucket trough the hole at the exact moment in which the hole is open.
e) ( 6 pts ) How far from the bucket will the fluid land (neglect the height of the scale)?


