P88
8.64

PHYSICS 8A(1) SAMPLE EXAM

The following is the second midterm from my Spring 2000 Physics 8A course. However, this exam covered Chapters 9–15 of RHW, while our April 15 exam will cover Chapters 8–14. There are three questions on this exam which cover fluids, and a knowledge of potential energy and conservation of energy was assumed on this exam. Solutions are included. The class average on this exam was 62%.

R. Dalven
8 April 2002

PHYSICS 8A - Dr. Dalven
Lec 2(TuTh)
Midterm 2
Thur., April 13, 2000, 11:10-12:30 pm

NAME : ______________________

SID # : ______________________

Disc #: ____________ Disc Day & Time: ____________

You may use one sheet no larger than 8.5" by 11" as a memory aid. As always, you must show your work. CORRECT ANSWERS WITH NO SUPPORTING WORK OR EXPLANATION RECEIVE NO CREDIT!!

1	/10
2	/25
3	/10
4	/15
5	/20
6	/20
TOTAL	/100

** Remember to give UNITS in your answers wherever appropriate.

NOTE:

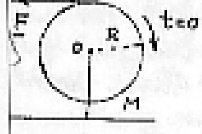
$$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$$

8A MT 2 SP 2000 (1)

(10)(1) The distance between the carbon atom (mass 2.00×10^{-26} kg) and the oxygen atom (mass 2.66×10^{-26} kg) in a CO molecule is 1.13×10^{-10} meters. (a) Calculate the location of the center of mass of the molecule, as measured from the carbon atom. (Assume that the constituent atoms do not vibrate.) (b) If the CO molecule translates (without rotating or vibrating) at a speed of 2000 m sec^{-1}, calculate the linear momentum of the molecule. [Part (a) = 7, (b) = 3]

8A MT 2 SPRING 2000 (2)

(25)(2) A cylindrical disc of mass $M = 10$ kg and radius $R = 2$ meters can rotate about an axis through its center O; the axis is normal to the plane of the paper. The disc is supported (as shown) on a horizontal surface.



At time $t = 0$, the disc is rotating clockwise at an angular speed of $\boxed{10}$ revolutions per second, when a force F of 100 Newtons, tangential to the disc, is applied. $\omega_0 = 10\ s^{-1}$ (a) Calculate the rotational inertia (moment of inertia) of the disc relative to the axis of rotation; (b) Calculate the magnitude of the torque applied to the disc (relative to the same axis) due to the force applied; (c) Calculate the angular acceleration due to this torque; (d) Is the angular speed in (c) increasing or decreasing? Justify your answer.; (e) Calculate the time required for the disc to come to rest; (f) Calculate the number of revolutions the disc makes in coming to rest. [a = b = c = d = e = 4 points, f = 5 points]

(10)(3) The deepest part of the Pacific Ocean is the "Marianas Trench" (near the island of Guam), where the water is 10^4 meters deep. (a) Assuming that sea water is incompressible with a mass density of 10^3 kg m^{-3}, calculate the absolute hydrostatic pressure at a depth of 10^4 m.; (b) A deep-diving submarine has a hatch on its top surface with a surface area of 1 m^2. Calculate the force exerted on the hatch by the water at a depth of 10^4 meters; (c) The force calculated in (b) has the same magnitude as the gravitational force by the Earth on a mass of M kilograms at a distance of 10^7 meters from the center of the Earth. Calculate the value of M; the mass of the Earth $M_E = 6 \times 10^{24}$ kg. [a = 5 pts, b = 2, c = 3]

(15)(4) It is an experimental fact that the absolute atmospheric pressure p varies with distance y above the Earth's surface as

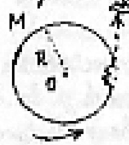
$$p(y) = p_0 e^{-Agy} \qquad (1)$$

where p_0 is the atmospheric pressure at the surface ($y=0$) of the Earth, A is a constant, and g is the acceleration (9.8 m sec^{-2}) due to gravity. The height y is measured positively in the upward direction. (a) Show that expression (1) satisfies the equation

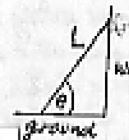
$$(dp/dy) = -\rho g ,$$

assuming that the mass density ρ of the air is proportional to the pressure p with constant of proportionality A; If $p = 0.5\,p_0$ at $y = 10^4$ meters, determine the numerical value of the constant A. [a = 10, b = 5]

(20)(5) A uniform cylindrical disc of mass M and radius R rotates about an axis through its center O with angular speed ω_0 as shown. The axis of rotation is normal to the plane of the paper. (a) Calculate the rotational kinetic energy K of the disc; (b) Calculate the angular momentum L of the disc; (c) At a time we will call $t = 0$, a chip of very small mass breaks off the edge of the disc and rises vertically above the point at which the chip broke off, as shown at left. Calculate the height h (above its starting point) to which the chip rises; (d) If $R = 0.1$ meter and ω_0 is 600 revolutions per minute, calculate the numerical value of h. [Parts (a), (b), (d) = 3 points each; (c) = 11 points]



(20)(6) A uniform ladder of mass m and length L leans at an angle θ against a frictionless wall. The coefficient of static friction between the ladder and the ground is μ. Calculate the smallest value of θ for which the ladder will not slip. Justify the fact that your answer is indeed the smallest value of θ for which the ladder does not slip.



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8A (Sec 2) Midterm #2 Solutions (Spring 2000)

(1)(a) Take the x-axis along the C-O axis of the CO molecule. Then the coordinate x_{cm} of the center of mass is given by

$$x_{cm} = \frac{m_O x_O + m_C x_C}{(m_O + m_C)}$$

If we take the origin at the C atom, $x_C = 0$ and we have

$$x_{cm} = \frac{m_O x_O}{(m_O + m_C)} = \frac{(2.66 \times 10^{-26})(1.13 \times 10^{-10})}{(2.66 + 2.00) \times 10^{-26}} \text{ meters}$$

since the C-O distance is 1.13×10^{-10} meters. Then

$$\boxed{x_{cm} = 0.645 \times 10^{-10} \text{ meters}} = 0.645 \text{ Å}$$

as measured from the C atom.

(b) Total mass M of the CO molecule is

$$M = (2.66 + 2.00) \times 10^{-26} = 4.66 \times 10^{-26} \text{ kg}$$

The momentum of the molecule P is given by

$$P = Mv = (4.66 \times 10^{-26})(2 \times 10^{3}) \text{ kg m sec}^{-1}$$

$$\boxed{P = 9.32 \times 10^{-23} \text{ kg m sec}^{-1}}$$

SOLUTIONS TO 8A MT 2, SP 2000 2

(2)(a) The rotational inertia of a cylinder with respect to an axis through its center is

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(10)(2)^2 = \boxed{20 \text{ kg}\cdot\text{m}^2}$$

(b) The torque $|\underline{\tau}| = |(\underline{r} \times \underline{F})| = rF$ since $\underline{r}$ is normal to $\underline{F}$ because $\underline{r}$ is radial and $\underline{F}$ is tangential. With $|\underline{r}| = R$,

$$\tau = RF = (2)(100) = \boxed{200 \text{ Newton-meters}}$$

(c) Since Newton's Second Law in rotational variables is

$$\tau = I\alpha \implies \alpha = (\tau / I) = (200/20) = 10 \text{ rad/sec}^2$$

$$\boxed{\alpha = 10 \text{ rad sec}^{-2}}$$

(d) The angular speed is <u>decreasing</u> because, from the figure in the problem, the direction of the applied force $\underline{F}$ is <u>opposite</u> to the direction (CW) of rotation.

(e) We know the <u>initial</u> angular speed $\omega_0 = 10$ revolutions/sec. Since <u>1 revolution is 2π radians</u>, $\omega_0 = 20\pi$ radians/sec. Also, at time t, the angular speed $\omega(t)$ is given by

$$\omega(t) = \omega_0 - \alpha t$$

where α is negative here because the disc is slowing down. We find the time t at which $\omega = 0$ (disc at rest) from

$$0 = \omega_0 - \alpha t = 20\pi - 10t \implies \boxed{t = 2\pi = 6.28 \text{ sec}}$$

(f) The number of revolutions is found from $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ where $(\theta - \theta_0)$ is the total angular displacement of the disc:

$$0 = \omega_0^2 + 2\alpha(\theta - \theta_0) = (20\pi)^2 - 2(10)(\theta - \theta_0) \implies \boxed{(\theta - \theta_0) = 20\pi^2 \text{ rad}}$$

Since $(\theta - \theta_0) = 20\pi^2$ radians, $\boxed{\# \text{ revolutions} = 10\pi} = 31.4$

(3) We know that the hydrostatic pressure p at depth y is given by

$$p(y) = p_0 + \rho g y$$

where ρ is the mass density (10^3 kg m^{-3}) of sea water, g is 9.8 m sec^{-2} and y is the depth (10^4 meters), while p_0 is the atmospheric (air) pressure over the ocean (1.0×10^5 Pa). Hence

$$p(10^4\,\text{m.}) = (10^5) + (10^3)(9.8)(10^4)\ \text{Pa}$$

$$\boxed{p(10^4\,\text{m.}) = 9.81 \times 10^7\ \text{Pa}}\ (\doteq 980\ \text{atmospheres})$$

(b) The force F on the hatch is given by $F = pA$, where p is the hydrostatic pressure and A is the surface area of the hatch. Then, since 1 Pa = 1 N m^{-2}, with $A = 1$ m^2,

$$\boxed{F = pA = (9.81 \times 10^7)(1) = 9.81 \times 10^7\ \text{Newtons}}$$

(c) The gravitational force F_g between Earth and mass M is

$$F = (G M_e M / r^2)$$

where r is the distance from the center of the Earth to mass M. Here $r = 10^7$ meters, so M is outside the Earth, and, although it is not necessary to state it in solving the problem, all of the Earth's mass $M_e = 6 \times 10^{24}$ kg acts as if it were concentrated at the center of the Earth. Then

$$9.81 \times 10^7 = [(6.67 \times 10^{-11})(6 \times 10^{24}) M / (10^7)^2]$$

Solving for M gives $\boxed{M = 2.4 \times 10^7\ \text{kg}}$

(4)(a) We want to show that $p(y) = p_0 \exp[-Agy]$ satisfies $(dp/dy) = -\rho g$ where $\rho = Ap$. Then

$$\frac{dp}{dy} = \frac{d}{dy}\left[p_0 e^{-Agy}\right] = p_0(-Ag)e^{-Agy} = p_0(-Ag)(p/p_0)$$

since $(p/p_0) = e^{-Agy}$. Thus

$$\frac{dp}{dy} = -Agp = -(Ap)g = -\rho g$$

so $p(y) = p_0 e^{-Agy}$ satisfies $(dp/dy) = -\rho g$ when $\rho = Ap$.

(b) To determine the numerical value of the constant A, we know that $p = 0.5 p_0$ when $y = 10^4$ meters. Since

$$\frac{p}{p_0} = e^{-Agy} \Rightarrow \ln(p/p_0) = -Agy$$

so $\ln(0.5) = -A(9.8)(10^4)$

$$-0.693 = -(9.8 \times 10^4)A$$

$$\boxed{A = 7.07 \times 10^{-6}\ \text{m}^{-2}\ \text{sec}^2}$$

We can see that the units (m^{-2} sec^2) are correct for A because

$$A = (\rho/p) = \frac{\text{kg m}^{-3}}{\text{N m}^{-2}} = \frac{\text{kg m}^{-3}}{\text{kg m sec}^{-2}\text{m}^{-2}} = \frac{\text{m}^{-3}}{\text{m}^{-1}\text{sec}^{-2}} = \text{m}^{-2}\text{sec}^2$$

(5)(a) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_0^2 = \frac{1}{4}MR^2\omega_0^2$

(b) $L = I\omega = \left(\frac{1}{2}MR^2\right)\omega_0 = \frac{1}{2}MR^2\omega_0$

(c) There are several ways to do this part of the problem, all of which are equivalent and all of which give the same answer. The chip flies upward vertically (under the influence of gravity) and has an initial upward velocity equal to its tangential velocity: $(R\omega_0) = v_0$.

The first method is to use conservation of kinetic plus potential energy since the only force acting on the chip is gravity, a conservative force. Calling $y=0$ the initial position of the chip and $y=h$ its final position (at which its vertical velocity is zero), we have

$$K(y=0) + U(y=0) = K(y=h) + U(y=h)$$

where U is the gravitational potential energy of the chip, which we take to be zero when $y=0$: $U(y=0) \equiv 0$. Then

$$\frac{1}{2}mv_0^2 + 0 = 0 + mgh \qquad [m = \text{chip mass}]$$
$$v_0^2 = 2gh$$
$$(R\omega_0)^2 = 2gh \implies \boxed{h = (R^2\omega_0^2/2g)}$$

The second method is to use the expression

$$v^2 = v_0^2 + 2a(y-y_0)$$

for velocity v in terms of initial velocity v_0, acceleration a, and displacement $(y-y_0)$ in the vertical direction.

(continued)

(5)(c)[continued] Here $(y-y_0) = h$, and $a = -g$, so at height h (when $v=0$) we have

$$0 = v_0^2 - 2gh$$
$$0 = R^2\omega_0^2 - 2gh \implies \boxed{h = (R^2\omega_0^2/2g)}$$

The third method is to use kinematics to find the time t for the chip to reach height h and use the value of the time to calculate the displacement h in the vertical direction. Then the velocity $v(t)$ of the chip at time t is given by

$$v(t) = v_0 - gt = R\omega_0 - gt$$

At height h, $v=0$ and the time $\boxed{t = (R\omega_0/g)}$. Then the height h is given by the projectile relation

$$y(t) = y_0 + v_0 t - \frac{1}{2}gt^2$$

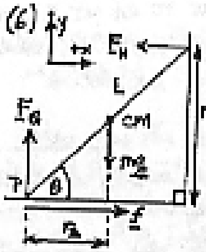
When $y=h$ (taking $y_0=0$ at the initial position of the chip), we have

$$h = 0 + (R\omega_0)(R\omega_0/g) - \frac{1}{2}g(R\omega_0/g)^2$$
$$h = \frac{R^2\omega_0^2}{g} - \frac{R^2\omega_0^2}{2g}$$
$$\boxed{h = (R^2\omega_0^2/2g)}$$

Any of these methods is a correct solution to (5)(c).

(d) If $R = 0.1$ m, $\omega_0 = 600$ rpm $= 10$ rps $= 20\pi$ radians/sec, then $\underline{h = 2.01 \text{ meters}}$.

(6)



$\underline{F}_N$ = normal force exerted on ladder by wall
$\underline{F}_G$ = normal force exerted on ladder by ground
$\underline{f}$ = frictional force exerted on ladder by ground
$m\underline{g}$ = weight of ladder (exerted at cm).

Then the frictional force $f \le \mu F_G$, since $\underline{f}$ acts only on the end of the ladder in contact with the ground. When the ladder is just about to slip, we have

$$f = \mu F_G .$$

Since the ladder is not slipping, the ladder is in <u>static equilibrium</u>, so that we have for translational equilibrium:

$$\sum \underline{F}_x = \underline{f} - \underline{F}_N = \mu F_G \hat{i} - F_N \hat{i} = (\mu F_G - F_N)\hat{i} = 0 \Rightarrow \boxed{\mu F_G = F_N} \quad (1)$$

$$\sum \underline{F}_y = \underline{F}_G - m\underline{g} = F_G \hat{j} - mg\hat{j} = (F_G - mg)\hat{j} = 0 \Rightarrow \boxed{F_G = mg} \quad (2)$$

The ladder is also in <u>rotational</u> static equilibrium, so, taking torques about point P above, we have, since cm is at center of ladder,

$$\tau_1 = F_N r_1 = F_N (L \sin\theta) \; ; \; \tau_2 = mg r_2 = mg\left(\tfrac{L}{2}\right)\cos\theta$$

and $\sum \tau = 0 = F_N L \sin\theta - mg\left(\frac{L}{2}\right)\cos\theta$ (3)

since the torques τ_1 and τ_2 act in opposite directions. From (1) and (2), $F_N = \mu mg$, so (3) becomes for $\sum \tau$,

$$0 = \mu mgL \sin\theta - mg\left(\tfrac{L}{2}\right)\cos\theta \Rightarrow \mu \sin\theta = \tfrac{1}{2}\cos\theta$$

so $\boxed{\tan\theta = (1/2\mu)}$ or $\boxed{\theta = \tan^{-1}(1/2\mu)}$

(continued →)

(6) [continued] for the smallest value of the angle θ for which the ladder will not slip and the ladder remain in static equilibrium, both translational and rotational.

We can see that the value $\theta = \tan^{-1}(1/2\mu)$ is the <u>smallest</u> value of θ for which the ladder is in static equilibrium as follows. If the angle θ is <u>less</u> than $[\tan^{-1}(1/2\mu)]$, i.e.,

$$\theta < \tan^{-1}(1/2\mu)$$

the sum $\sum \tau$ of the torques

$$\sum \tau = (\mu mgL \sin\theta - mg(\tfrac{L}{2})\cos\theta) \neq 0$$

and the ladder (not in equilibrium) slips. If θ is less than $[\tan^{-1}(1/2\mu)]$, that means

$$\frac{\sin\theta}{\cos\theta} < \frac{1}{2\mu} \Rightarrow \sin\theta < \left(\frac{1}{2\mu}\right)\cos\theta$$

and the $\sum \tau$ of the torques becomes, with

$$\mu mgL \sin\theta < \left(\tfrac{1}{2}\right) mgL \cos\theta$$

and so $\sum \tau < 0$, meaning (with the signs for torques used above) that the ladder moves with the cm rotating clockwise and the ladder falls to the ground. (We know from everyday experience that, for values of θ <u>larger</u> than $[\tan^{-1}(1/2\mu)]$ the ladder does not slip.)