UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME132 Dynamic Systems and Feedback

Midterm Examination I **Spring 2007**

Closed Book and Closed Notes. Two 8.5×11 pages of handwritten notes allowed.

Your Name:

Please answer all questions and draw a box around your final answer for each question.

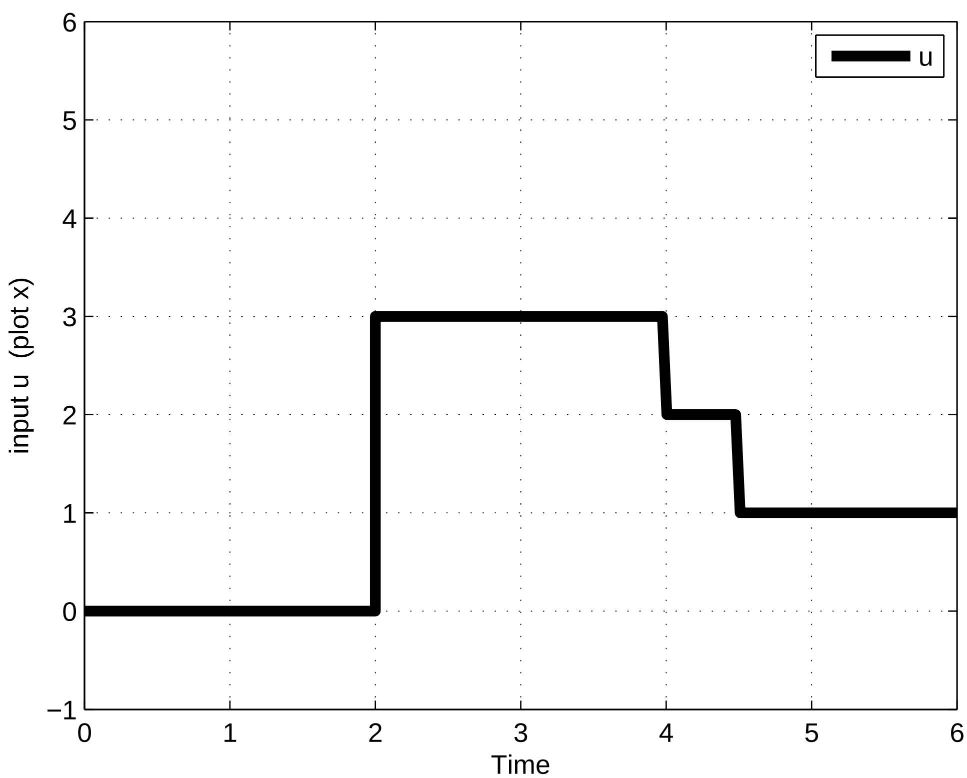
Problem:	1	2	3	Total
Max. Grade:	20	40	40	100
Grade:				

1 Problem

Neatly and accurately sketch, on the figure below, the solution $x(t)$ for $t \geq 0$ to the differential equation

$$\dot{x}(t) = -3\,x(t) + 6\,u(t)$$

subject to the initial condition $x(0) = -1$ and the forcing function $u(t)$, which is plotted in the figure below.



2 Problem

Let $y_p(t)$ be a particular solution for the following first order ODE

$$\dot{y}(t) + ay(t) = b\,u(t) \tag{1}$$

and assume that $a > 0$ and $b > 0$ are real constants.

1) Assume that the input $u(t)$ is expressed as

$$u(t) = u_{_R}(t) + j\,u_{_I}(t),$$

   where $j^2 = -1$ and $u_R(t)$ and $u_I(t)$ are real functions of time.

   If a particular solution $y_p(t)$ for the above input is expressed as

$$y_p(t) = y_{p_R}(t) + j\,y_{p_I}(t),$$

   write down the differential equations that $y_{p_R}(t)$ and $y_{p_I}(t)$ respectively satisfy.

2) Assume that

$$u(t) = e^{j\omega t},$$

   where $\omega > 0$ is a real constant.

   (a) A particular solution $y_p(t)$ for this input can be written as

$$y_p(t) = G(\omega)\,u(t)\,. \tag{2}$$

   Determine an expression for the complex number $G(\omega)$ in terms of the parameters a, b and ω.

   (continues on the next page)

(b) The particular solution $y_p(t)$ in Eq. (2) can also be written as

$$y_p(t) = M(\omega)\, e^{(j\omega t + \phi(\omega))} .$$

where $M(\omega) \geq 0$ and $\phi(\omega)$ are real. Obtain expressions for $M(\omega)$ and $\phi(\omega)$ in terms of the parameters a, b and ω.

3) Assume that

$$u(t) = \cos(\omega t) = \text{Real}\left(e^{j\omega t}\right) . \tag{3}$$

where $\omega > 0$ is a real constant.

(a) A particular solution $y_p(t)$ for this input can be written as

$$y_p(t) = Y(\omega)\, \cos\left(\omega t + \phi(\omega)\right) . \tag{4}$$

Determine expressions for $Y(\omega)$ and $\phi(\omega)$ in terms of a, b and ω.

(b) Fig. 1 shows the response of the ODE in Eq. (1) when the input $u(t)$ is given by Eq. (3) and $y(0) = 0$. A steady state response is achieved for $t \geq 1$.

Determine the values of the parameters ω, a and b.

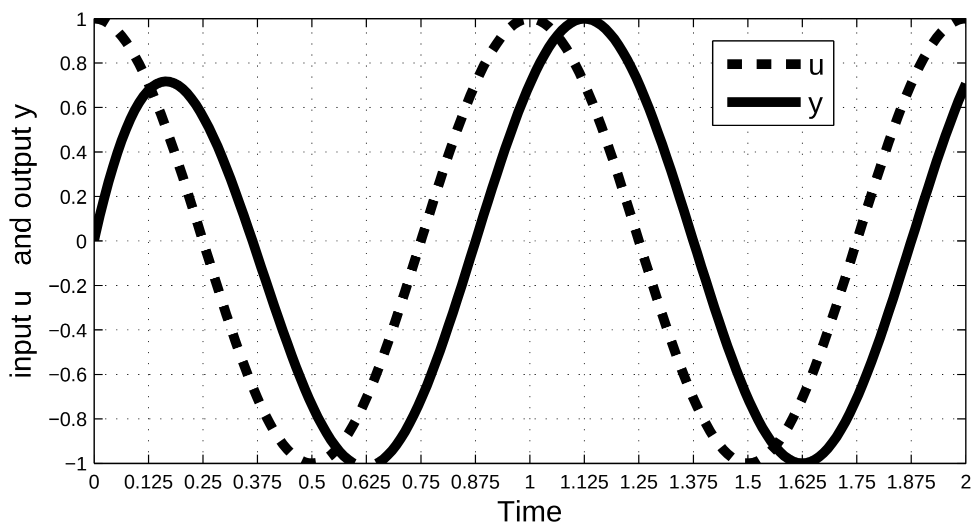


Figure 1: Response $y(t)$ for a cosine input function.

3 Problem

The ODE which describes the angular rotation of a D.C. motor is

$$\ddot{\theta}(t) + \alpha\,\dot{\theta}(t) = E\,u(t) + G\,d(t) \tag{1}$$

where

- $\theta(t)$ is dc-motor's angular rotation and $\dot{\theta}(t)$ and $\ddot{\theta}(t)$ are its angular velocity and acceleration respectively
- $u(t)$ is the control input and d is the disturbance input
- $E = 1$, $G = 1$ and $\alpha = 0.1$ are constants of appropriate dimensions.

The control objective is that the dc-motor output $\theta(t)$ converges to a **constant** steady state value $\bar{\theta}_{des}$.

1) Assume that $\theta(0) \neq \bar{\theta}_{des}$.

   Is it possible to achieve the control objective with a constant **open loop** control input $\bar{u}_{op}$, even if $d(t) = \bar{d}$ is constant and is perfectly known? Explain why or why not.

2) Assume that the closed loop control is

$$u(t) = K_P\,[\theta_{des} - \theta(t)] - K_D\,\dot{\theta}(t) \tag{2}$$

   where K_P and K_D are control gains.

   The control system block diagram is shown in Fig. 2.

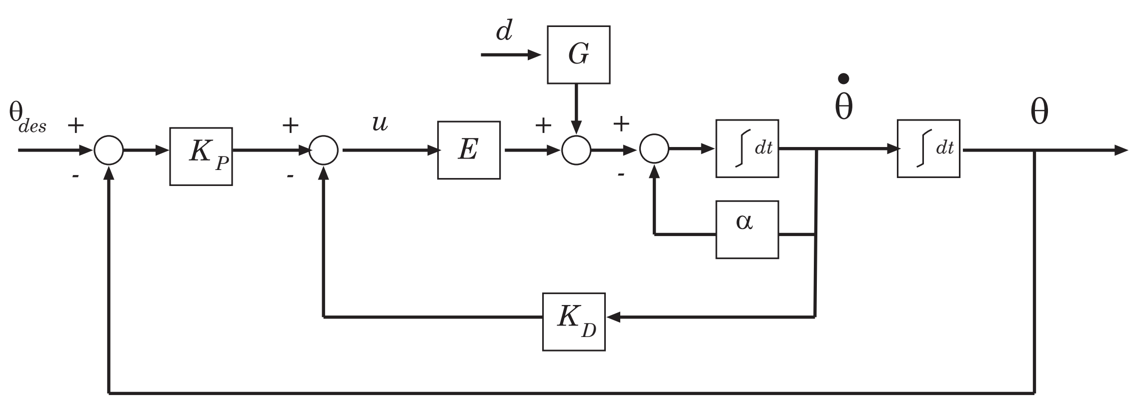


Figure 2: DC motor control systems

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(a) Derive the second order ODE which describes the closed loop system. Notice that the system has two inputs: θ_{des} and $d(t)$ and one output: $\theta(t)$.

(b) Determine the conditions that the control gains K_P and K_D must satisfy so that the homogeneous solution of the close loop system is asymptotically stable.

(c) Assume that that the desired angular velocity is a constant, and the disturbance is zero, i.e.

$$d(t) = 0 \qquad\qquad \theta_{des}(t) = \bar{\theta}_{des}$$

and that K_P and K_D are selected so that the homogeneous solution of the close loop system is asymptotically stable. Determine the steady state angular rotation $\bar{\theta}$.

(d) Calculate the required control gains K_p and K_d so that the homogeneous solution of the closed loop system will have a damping ratio $\xi = 0.707$ and a natural frequency $\omega_n = 10$ rad/sec.

(e) Draw a sketch of the close loop system response under the conditions outlined in items (c) and (d), and under the initial conditions

$$\theta(0) = \dot{\theta}(0) = 0\,.$$