Problem 1. (90 points)  Let $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ be two finite automata over a common alphabet.

a. Construct a finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ such that $L(M)$ contains precisely the words in $L(M_1)$ which have a suffix in $L(M_2)$.

b. Apply your construction from part (a) to the following two automata.

![Diagram](image1)

M1: 0 1 0.1
A B
M2: α 1
β 0 0 1

c. Is the result from part (b) deterministic? If not, use the subset construction to determinize it.

d. Is the result from part (c) minimal? If not, use the minimization algorithm to minimize it.

e. Let $A$ be the language of the automata from parts (b)–(c). What is the index of $A$? Describe each $\equiv_A$-equivalence class by a regular expression.

You can double check your result by thinking about which languages are accepted by the automata of part (b), and which language results from applying the operation of part (a) to these languages.

Problem 2. (60 points)  For the two languages $B_1$ and $B_2$ defined below do the following.

If $B_i$ is regular: Give a finite automaton that recognizes $B_i$.

If $B_i$ is not regular: Give both a proof that $B_i$ is not regular (use the pumping lemma or the fact that $\{0^n1^n : n \geq 0\}$ is not regular) and a pushdown automaton that recognizes $B_i$.

a. $B_1$ is the set of all words in $\{0, 1\}^*$ that contain an equal number of occurrences of 00 and 11. (Count overlapping occurrences; for example, 110001111 contains 2 occurrences of 00 and 4 occurrences of 11.)

b. $B_2$ is the set of all words in $\{0, 1\}^*$ that contain an equal number of occurrences of 01 and 10.

Before you begin to answer the questions, think about which words are in $B_1$ and $B_2$, and which are not, by listing a few examples.