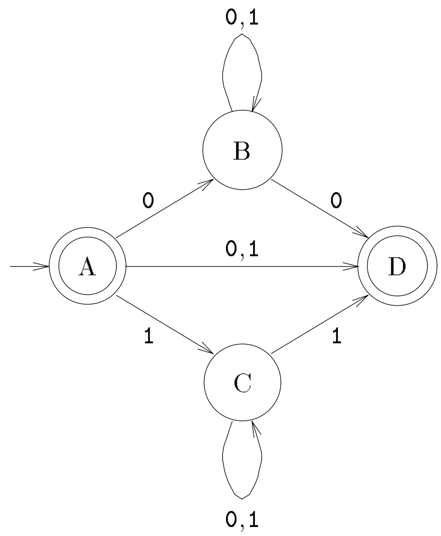
CS 172 — Spring 1997	**Prelim 1**
Computability and Complexity	**March 3, 1997**

Problem 1. (30 points)

a. Describe in words the language accepted by the following NFA:



b. Write a regular expression that accepts the same language. (You need not justify how you obtain the expression.)

c. Determinize the automaton using the subset construction. (Label each DFA state with the corresponding set of NFA states).

Problem 2. (30 points)

a. The function f on regular expressions is defined inductively:

$$
\begin{aligned}
f(\emptyset) &= \infty \\
f(a) &= 1 \quad \text{for all } a \in \Sigma \\
f(R_1 \cup R_2) &= \min(f(R_1), f(R_2)) \\
f(R_1 \circ R_2) &= f(R_1) + f(R_2) \\
f(R^*) &= 0
\end{aligned}
$$

Given a regular expression R, what does $f(R)$ compute?

b. Write an inductive function g so that $g(R)$ computes the set of all first letters of strings in the language $L(R)$. For example,

$$g((a \cup bc)^* d) = \{a, b, d\}.$$

(You may use f in the definition of g.)

c. If R is a regular expression with n symbols, how expensive is the computation of $f(R)$ in O-notation? (Give a brief justification for your answer.)

Problem 3. (30 points)
For two strings $x, y \in \Sigma^*$, we write $x\#y$ for the string that alternates letters from x with letters from y:

$$\begin{aligned} x \# \varepsilon &= x \\ \varepsilon \# y &= y \\ ax \# by &= ab(x\#y) \quad \text{for all } a, b \in \Sigma. \end{aligned}$$

For example,

$$cal \# bears = cbaelars.$$

For two languages $A, B \subseteq \Sigma^*$, let

$$A\#B = \{x\#y \mid x \in A \text{ and } y \in B\}.$$

Given a finite automaton $(Q_A, \Sigma, \delta_A, q_A, F_A)$ that accepts A, and a finite automaton $(Q_B, \Sigma, \delta_B, q_B, F_B)$ that accepts B, construct a finite automaton that accepts $A\#B$. (You need not justify your construction.)

Problem 4. (30 points)
For a string $x \in \Sigma^*$, we write $[x]$ for the set of all anagrams of x (an anagram is a rearrangement of the letters of a word). For example,

$$[cal] = \{cal, cla, acl, alc, lca, lac\}.$$

For a language $A \subseteq \Sigma^*$, let

$$[A] = \{x \mid x \in [y] \text{ for some } y \in A\}.$$

a. Find two regular languages B and C such that $[B] \cap C = \{\mathtt{0}^n\mathtt{1}^n \mid n \geq 0\}$.

b. Use the pumping lemma to show that the language $[B]$ is not regular.

Problem 5. (30 points)
Consider the language $A_k = (\mathtt{0} \cup \mathtt{1})^*\mathtt{0}(\mathtt{0} \cup \mathtt{1})^{k-1}$, where $k \geq 1$ is an arbitrary integer.

a. Describe an NFA with $k+1$ states that accepts A_k.

b. Find 2^k strings in $\{\mathtt{0}, \mathtt{1}\}^*$ such that no two of the strings are A_k-equivalent. (Justify your answer.)

c. What can you conclude about the number of states of any DFA that accepts A_k? (Justify your answer.)