Problem 1. (50 points) Determine which of the following two languages are context-free:

\[
A = \{ w \in \{0,1\}^* : w = w^R \}, \\
B = \{ w \in \{0,1\}^* : w \neq w^R \}.
\]

Here, \(w^R\) is the word \(w\) reversed. Prove your answers.

Problem 2. (100 points)

a. Consider a machine \(M\) and a state \(q\) of \(M\). The state \(q\) is dead if for all input words \(w\) and all runs \(r\) of \(M\) on input \(w\), the state \(q\) does not occur in \(r\). The state \(q\) is redundant if \(L(M) = L(M\backslash q)\), where \(M\backslash q\) is the machine that results from \(M\) by removing the state \(q\), as well as all transitions in and out of \(q\).

If \(q\) is dead, does it follow that \(q\) is redundant?
If \(q\) is redundant, does it follow that \(q\) is dead?

b. Given a machine \(M\) and a state \(q\) of \(M\), the dead-state problem asks if \(q\) is a dead state of \(M\).
Given a machine \(M\) and a state \(q\) of \(M\), the redundant-state problem asks if \(q\) is a redundant state of \(M\).

Consider the following six problems:

\[
D_{\text{NFA}}, D_{\text{PDA}}, D_{\text{TM}}: \text{the dead-state problems for NFAs, PDAs, and TMs.} \\
R_{\text{NFA}}, R_{\text{PDA}}, R_{\text{TM}}: \text{the redundant-state problems for NFAs, PDAs, and TMs.}
\]

For each of these six problems, determine which of the following four statements is true:

S1 The problem is recursive.
S2 The problem is r.e., but not recursive.
S3 The problem is co-r.e., but not recursive.
S4 The problem is neither r.e. nor co-r.e.

Prove your answers. You can use the membership, emptiness, universality, and equivalence problems for NFAs, PDAs, and TMs, and what we learned about them in class.