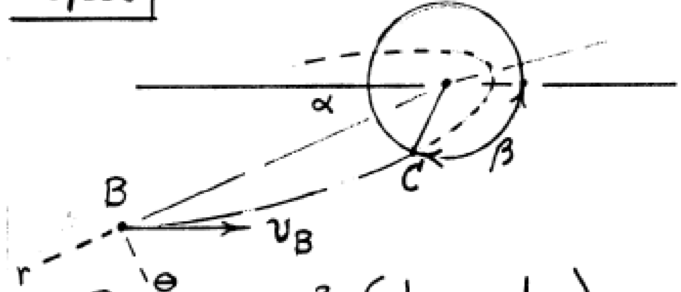
▶3/303



At B, $r = \sqrt{29}\,R$

$$\alpha = \tan^{-1}\left(\frac{2R}{5R}\right) = 21.8^\circ$$

$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$$

At B : $$3200^2 = 2(9.825)(6.371\times10^6)^2\left[\frac{1}{\sqrt{29}\,6.371(10^6)} - \frac{1}{2a}\right]$$

$$a = 3.066\times10^7 \text{ m}$$

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(3200)^2 = 5.120\times10^6 \text{ m}$$

$$V_B = -\frac{mgR^2}{r_B} = -m\frac{(9.825)(6.371\times10^6)^2}{\sqrt{29}\,(6.371\times10^6)} = -1.162\times10^7 \text{ m}$$

$$E = T_B + V_B = -6.504\times10^6 \text{ m}$$

$$v_\theta = 3200\sin\alpha = 1188.5 \text{ m/s}$$

$$h = rv_\theta = \sqrt{29}\,(6.371\times10^6)(1188.5) = 4.077\times10^{10} \text{ kg-m}^2\text{/s}$$

$$e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}}$$

$$e = \sqrt{1 + \frac{2(-6.504m)(4.077\times10^{10})^2}{m\,(9.825)^2\,(6.371\times10^6)^4}}$$

$$= 0.9295$$

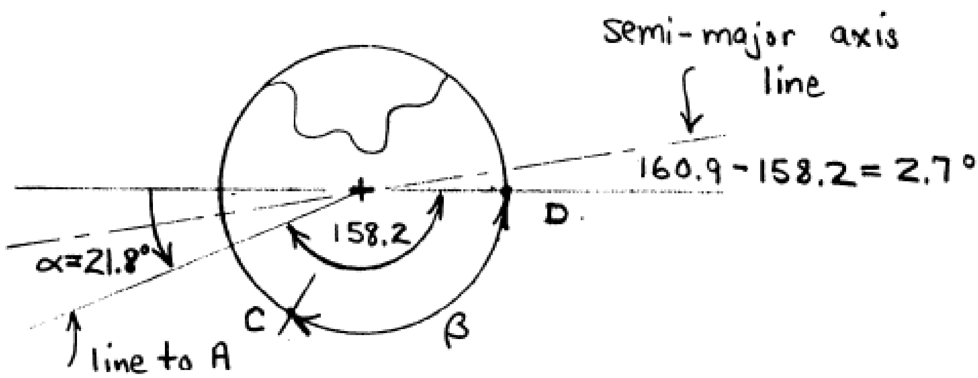
$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

At B : $$\sqrt{29}\,(6.371\times10^6) = \frac{(3.066\times10^7)(1-0.9295^2)}{1+0.9295\cos\theta}$$

$$\theta = 160.9^\circ$$

At C : $$6371(10^6) = \frac{(3.066\times10^7)(1-0.9295^2)}{1+e\cos\theta}$$

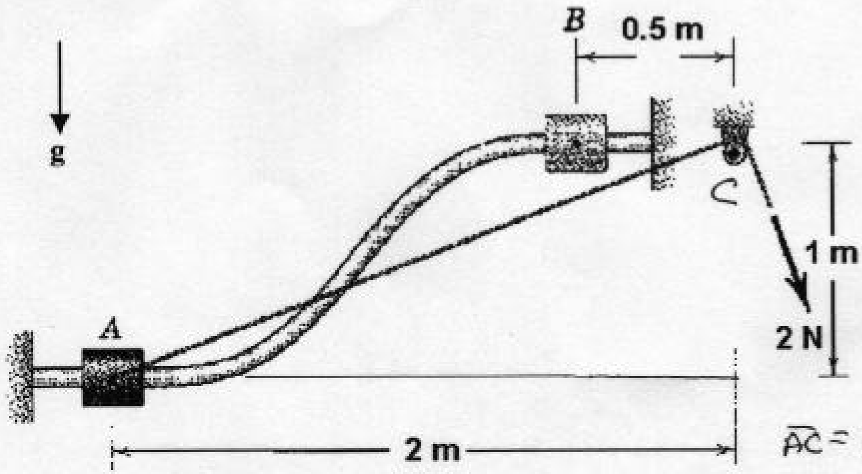
$$\theta = 111.8^\circ$$

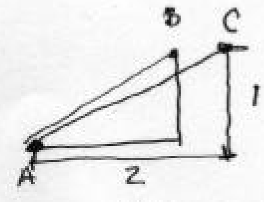


$$\beta = 111.8 - 2.7 = \underline{109.1^\circ}$$

Problem 2 (20 points)

The 0.5 kg slider moves freely along the fixed curved rod from *A* to *B* in the vertical plane under the action of the constant 2 N tension in the cord. If the slider is released from rest at *A*, calculate its velocity *v* as it reaches *B*.





$\overline{AC} = (2^2 + 1^2)^{1/2} = \sqrt{5}$

Goal: Determine the velocity of the slider as it approaches point B.

Given: Picture above. Frictionless path.

Draw: Above

Solve: Because there is no friction, energy will be conserved. The work done by gravity and tension will influence the velocity of the slider.

Initially the slider is at rest $V_0 = 0$

Initial Energy $= 0$ since we take potential energy datum at the ~~high~~ height of point A.

~~Final~~

Final: Slider has potential and kinetic energy $mgh + \frac{1}{2}mV^2$

Work done by tension $W = \int F\,dx = 2 \cdot (\sqrt{5} - 0.5) = 3.47\ J$

~~Work~~ Initial energy + Work = Final Energy

$$0 + 3.47 = mgh + \tfrac{1}{2}mV^2$$

$$V^2 = \frac{2(3.47 - mgh)}{m} = \frac{2(3.47 - 0.5 \cdot 9.8 \cdot 1)}{0.5} = -5.72 \left(\tfrac{m}{s}\right)^2$$

Because V^2 is negative, we observe that the work done by the rope is ~~no~~ insufficient to move the slider to B.

The slider never reaches point B.

3/344 D to E : $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$$-\rho = -\frac{1}{2} g t^2 , \quad t = \sqrt{\frac{2\rho}{g}}$$

$x = x_0 + v_{x_0} t$: $d = v_D \sqrt{\frac{2\rho}{g}}$, $v_D = d\sqrt{\frac{g}{2\rho}}$

A to D : $U = \Delta T$

$$\frac{1}{2} k \delta^2 - \mu_k m g \rho - m g \rho = \frac{1}{2} m \left(d^2 \frac{g}{2\rho}\right) - 0$$

$$\underline{\delta = \sqrt{\frac{mg}{k}} \sqrt{\frac{d^2}{2\rho} + 2\rho(1+\mu_k)}}$$

But speed at top of hill must be ≥ 0:

$U = \Delta T$: $\frac{1}{2} k \delta^2 - \mu_k m g \rho - 3 m g \rho = \frac{1}{2} m v^2 - 0 \geq 0$

or $\delta \geq \sqrt{\frac{2 m g \rho}{k} (3 + \mu_k)}$

$$\therefore \quad \frac{mg}{k}\left(\frac{d^2}{2\rho} + 2\rho[1+\mu_k]\right) \geq \frac{2 m g \rho}{k}(3 + \mu_k)$$

or $\underline{d \geq 2\sqrt{2}\,\rho}$

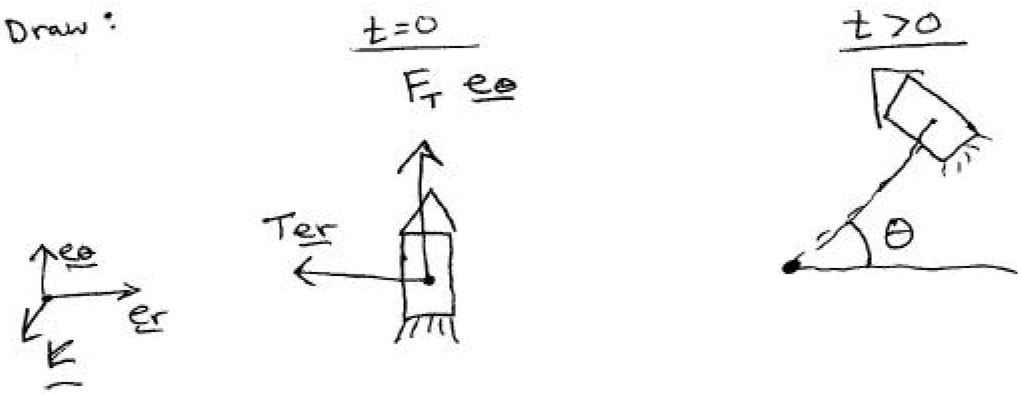
Problem 4 MT 2

Given: Tethered rocket, mass m, Length L, Constant thrust 12 N.

Assume: Neglect: gravity, friction
Rocket is initially at rest
Tether is inextensible.

Goal: Determine the angular momentum and velocity at $t = 3$ seconds.

Draw:



Solve: Conservation of Angular momentum about point O

$$H_0 + \int M\,dt = H_f \quad \text{at point } O$$

$$H(t=0) + \int_0^3 M\,dt = H(t=3)$$

Initially rocket at rest $\Rightarrow H(t=0) = 0$

$$M = \underline{r} \times \underline{F} = r\underline{e_r} \times (-T\underline{e_r} + F_T\underline{e_\theta}) = rF_T\underline{k} \quad = \text{constant}$$

$$\int_0^3 M\,dt = 3rF_T = (3)L(12) = 36L$$

$$\underline{H}(t=3) = \int_0^3 M\,dt = \boxed{36L\underline{k}}$$

Vector quantity
Direction is important

Definition of angular momentum with constant mass

$$H = \underline{r} \times m\underline{v} = L\underline{e_r} \times m\,V\underline{e_\theta} = mLV\underline{k}$$

$$mLV = 36L$$

$$V = \frac{36}{m} \qquad \boxed{\underline{V} = \frac{36}{m}\underline{e_\theta}}$$

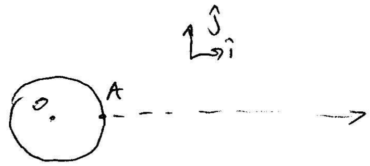
Vector quantity
Direction is important.

Problem 5

Goal: Find the escape velocity, v.

Given: Physical constants of earth : $R = 6371$ km, $M = 5.98 \times 10^{24}$ kg

Draw:



Assume: no friction.

solve:

FBD:

$$\underline{F_G} = \frac{G m_E m_s}{r^2} \hat{\imath} \qquad \underline{r} = r\hat{\imath}$$
$$\dot{\underline{r}} = \dot{r}\hat{\imath}$$

Use work-Energy theorem:

$$\dot{T} = F \cdot \underline{v}$$
$$= \left(\frac{-G m_E m_s}{r^2}\hat{\imath}\right) \cdot \dot{r}\hat{\imath}$$
$$\frac{d}{dt}\left(\tfrac{1}{2} m_s \underline{v} \cdot \underline{v}\right) = \frac{-G m_E m_s}{r^2}\dot{r} = \frac{-d}{dt}\left(\frac{-G m_E m_s}{r}\right)$$
$$\frac{d}{dt}\left(\tfrac{1}{2} m_s v^2 + \frac{-G m_E m_s}{r}\right) = 0 \implies \text{Energy is conserved}$$

$$\left(\tfrac{1}{2} v^2 + \frac{-G m_E}{r}\right)_A = \left(\tfrac{1}{2} v^2 + \frac{-G m_E}{r}\right)_B$$

To just escape the earth's gravity, the potential energy of the gravitational force must be zero. To find the minimum velocity, the final velocity must be as small as it can get (zero velocity).

$$\tfrac{1}{2} v^2 + \frac{-G m_E}{R} = 0$$
$$v^2 = \sqrt{2}\,\frac{2 G m_E}{R}$$
$$\underline{v \approx 11185 \text{ m/s}}$$