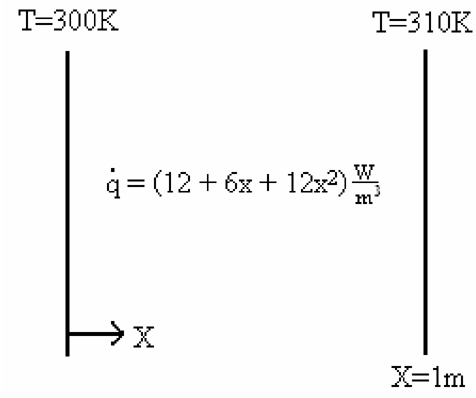
Problem 1 (40)



Goal: Find q total

Solution:

Fastest way

Total heat generated = Total heat lost = q

$$q = \int \dot{q}\, dV = A_c \int_0^1 \dot{q}\, dx$$

$$q = A_c \int_0^1 \left(12 + 6x + 12x^2\right) dx$$

$$q = 1\left[12x + 3x^2 + 4x^3\right]\Big|_0^1$$

$$q = 19W$$

Standard analytical way

$$k\frac{d^2T}{dx^2} + \dot{q} = 0$$

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{k} = -\frac{12 + 6x + 12x^2}{k}$$

$$\frac{dT}{dx} = -\frac{12x + 3x^2 + 4x^3}{k} + C_1$$

$$T(x) = -\frac{6x^2 + x^3 + x^4}{k} + C_1 x + C_2$$

Apply BC

$$300 = T(0) = C_2$$
$$310 = T(1) = -\frac{6+1+1}{1} + C_1 + 300$$
$$C_1 = 18$$
$$T(x) = -\frac{6x^2 + x^3 + x^4}{k} + 18x + 300$$
$$T(x) = -x^4 - x^3 - 6x^2 + 18x + 300$$

Calculate flux

$$q(x) = -kA\frac{dT}{dx} = -\left(-4x^3 - 3x^2 - 12x + 18\right) = 4x^3 + 3x^2 + 12x - 18$$
$$q_{total} = -q(0) + q(1) = 18 + (4 + 3 + 12 - 18)$$
$$q_{total} = 19W$$

Finite Difference method

1 node

T surf not usable

Entire wall is control volume, so q_net_in=-qgenerated

qgenerated – if evaluated as integral, exact answer

if evaluated at midpoint (12 + 6*(1/2) + 12*(1/4))=18W

2 node

If split the difference, T surf not usable

If one on each boundary, , not useful

3 node

Can solve for T_middle

$$q_{in} - q_{out} + q_{gen} = kA\frac{\left(300 - T_{middle}\right)}{0.5} - kA\frac{\left(T_{middle} - 310\right)}{0.5} + \frac{18}{2} = 0$$
$$2T_{middle} = 300 + 310 + \frac{18}{4}$$
$$T_{middle} = \frac{614.5}{2} = 307.25$$

Then look at each side

Left, note volume = 1/4

$$q_{in} - q_{out} + q_{gen} = 0$$
$$q_{in} = q_{out} - q_{gen} = kA\frac{300 - T_{middle}}{0.5} - \frac{12}{4}$$
$$q_{in} = \frac{300 - 307.25}{0.5} - 3 = -17.5$$

Right, note volume = 1/4

$$q_{in} - q_{out} + q_{gen} = 0$$

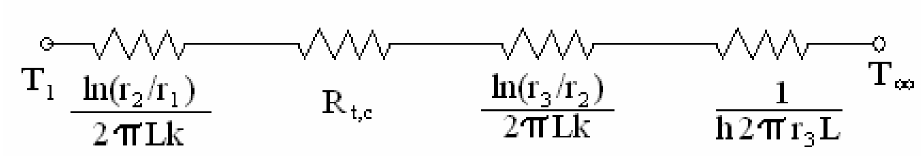
$$q_{out} = q_{in} + q_{gen} = kA\frac{307.25 - 310}{0.5} + \frac{30}{4} = 2$$

Sum

$$q_{out\_total} = q_{right\_out} - q_{left\_in} = 19.5W$$

Problem 2 (40)

Draw a circuit between inner radius and air

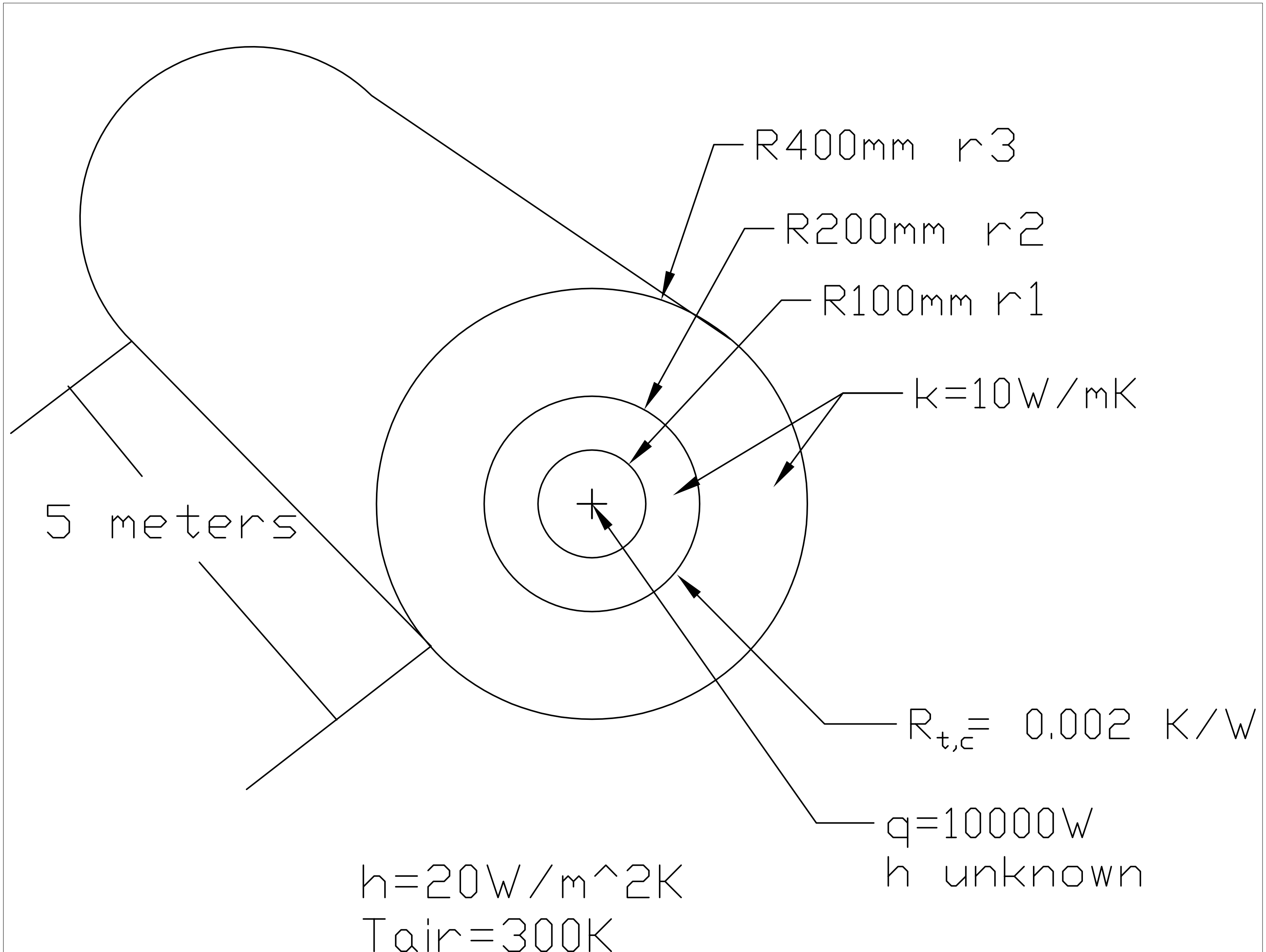


$$q = \frac{T_1 - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi Lk} + R_{t,c} + \frac{\ln(r_3/r_2)}{2\pi Lk} + \frac{1}{h2\pi r_3 L}}$$

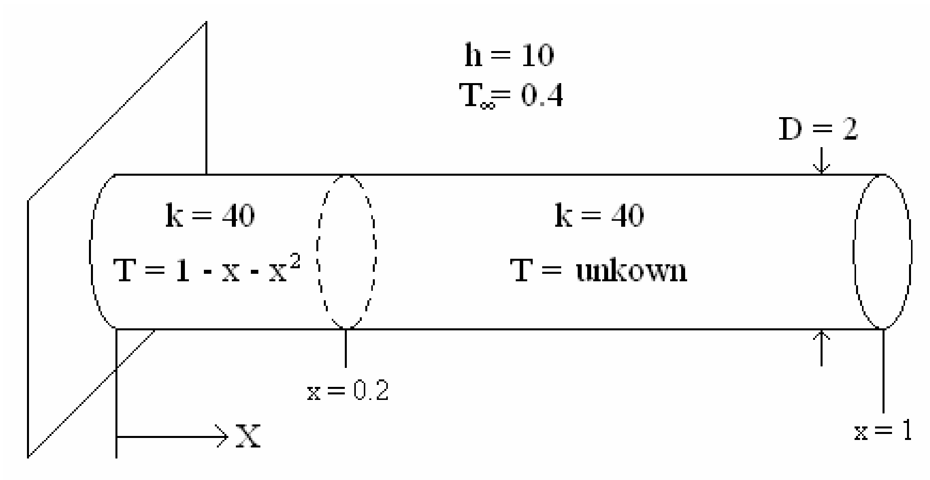
$$T_1 = q\left[\frac{\ln(r_2/r_1)}{2\pi Lk} + R_{t,c} + \frac{\ln(r_3/r_2)}{2\pi Lk} + \frac{1}{h2\pi r_3 L}\right] + T_\infty$$

$$T_1 = 10000W\left[\frac{\ln\left(\frac{0.2m}{0.1m}\right)}{2\pi(5m)\left(10\frac{W}{mK}\right)} + 0.002K/W + \frac{\ln\left(\frac{0.4m}{0.2m}\right)}{2\pi(5m)\left(10\frac{W}{mK}\right)} + \frac{1}{\left(20\frac{W}{m^2K}\right)2\pi(0.4m)(5m)}\right] + 300K$$

$T_1 = 403.916$ K


R400mm r3
R200mm r2
R100mm r1
k=10W/mK
5 meters
$R_{t,c}$= 0.002 K/W
q=10000W
h unknown
h=20W/m^2K
Tair=300K

Problem 3 (30)



Goal: Find the steady state heat loss from fin region x=0.2 to x=1

Solution

Drawing a control volume around x=0.2 to x=1, $q_{in} = q_{out}$.

$$q_{in} = -kA\left.\frac{dT}{dx}\right|_{x=0.2^+}$$

q should be continuous across the boundary

$$q_{in} = -kA\left.\frac{dT}{dx}\right|_{x=0.2^+} = -kA\left.\frac{dT}{dx}\right|_{x=0.2^-}$$

Plug in T

$$q_{in} = -k\left(\frac{\pi D^2}{4}\right)\left.\frac{d\left(1-x-x^2\right)}{dx}\right|_{x=0.2^-}$$

$$q_{in} = -\frac{k\pi D^2}{4}\left.\left(-1-2x\right)\right|_{x=0.2}$$

$$q_{in} = -\frac{k\pi D^2}{4}\left(-1-0.4\right)$$

$$q_{in} = \frac{1.4k\pi D^2}{4}$$

$$q_{in} = \frac{1.4\left(40\right)\pi\left(2\right)^2}{4} = 175.9$$

$$q_{out} = 175.9$$