ME 132, Fall 2003, Quiz 2

Name:

# 1	# 2	# 3	# 4	TOTAL
6	20	9	15	50

NOTE: Any unmarked summing junctions are positively signed (+).

1. The input u, and output y, of a single-input, single-output system are related by

$$y^{[3]}(t) + 6y^{[2]}(t) + 2y^{[1]}(t) + 3y(t) = 2u^{[2]}(t) - 5u^{[1]}(t) - 5u(t)$$

   (a) Find the transfer function from U to Y

   (b) Show that this is a stable system.

   (c) If $u(t) \equiv 2$ for all $t \geq 0$, what is the limiting value of y, namely $\lim_{t\to\infty}$?

   (d) Suppose the input is sinusoidal, $u(t) = \sin(100t)$. In the steady state, what is the <u>approximate</u> amplitude of the sinusoidal output y?

2. Two masses are connected by a linear, massless spring. An external force f is applied to Mass #1. Suppose that v_1 is the velocity of m_1, v_2 is the velocity of m_2 (both measured relative to the same inertial frame).

Since the spring is massless, the forces at each end must balance, and are equal to $k\Delta$, where Δ is the stretch in the spring (from its unstretched length L). This is shown above, right.

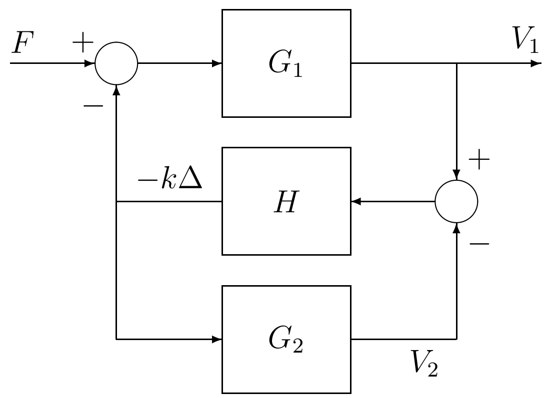
(a) Interpret the meaning of each of the equations below. Draw a Free-body diagram, or other appropriate picture to assist in your explanation.

i. $f(t) + k\Delta(t) = m_1\dot{v}_1(t)$

ii. $-k\Delta(t) = m_2\dot{v}_2(t)$

iii. $\dot{\Delta}(t) = v_2(t) - v_1(t)$

(b) Find transfer functions G_1, G_2 and H so that the block diagram below represents the relationships you wrote on the previous page. Note that a few internal signals ($-k\Delta$ and v_2) are marked. These are to be used to determine G_1, G_2 and H.

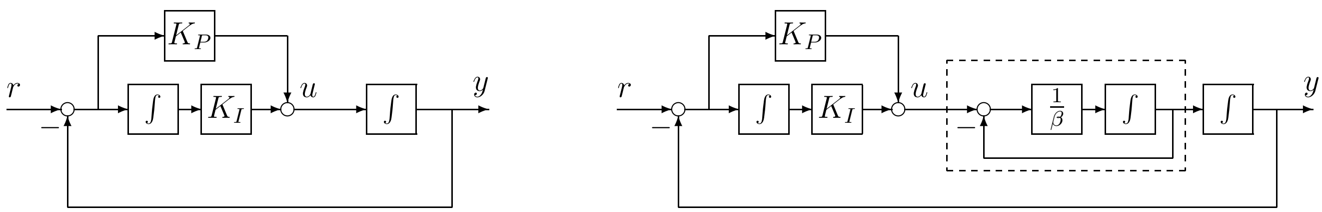


(c) Find the transfer function from F to V_1.

(d) Suppose that the goal is to control V_1. Relabel the velocity as Y, so $Y = V_1$. Consider a simple proportional control strategy, $F = K_P(R - Y)$, where R is a "desired velocity" reference signal, and K_P is a constant gain. Draw a simple block-diagram, and derive the closed-loop transfer function from $R \to Y$.

(e) What are the conditions on K_P for closed-loop stability?

3. Shown below are two systems. The system on the left is the **nominal system**, while the system on the right represents a deviation from the nominal (the insertion of the dashed box) and is called the **perturbed system**.



Based on the values of K_P an K_I, and some analysis, you should have a general idea of how the nominal system behaves (eg., the effect of r on u and y). Consider 3 different possibilities (listed below) regarding the relationship between the nominal and perturbed systems:

(a) The perturbed system behaves pretty much the same as the nominal system.

(b) The perturbed system behaves quite differently from the nominal system, but is still stable.

(c) The perturbed system is unstable.

For each row in the table below, which description from above applies? Write **a**, **b**, or **c** in each box. Show work below.

K_P	K_I	β	Your Answer
2.8	4	0.02	
1.4	1	1	
14	100	0.2	
70	2500	0.02	

4. A process, with input v and output y is governed by

$$\dot{y}(t) - 2y(t) = v(t)$$

(a) What is the transfer function from V to Y?

(b) Suppose $y(0) = 1$, and $v(t) \equiv 0$ for all $t \geq 0$. What is the solution $y(t)$ for $t \geq 0$. Is the process stable?

(c) Suppose that the input v is the sum of a control input u and a disturbance input d, so $v(t) = u(t) + d(t)$. Consider a PI control strategy, $u(t) = K_P [r(t) - y(t)] + K_I z(t)$, $\dot{z}(t) = r(t) - y(t)$. Draw a block diagram of the closed-loop system using transfer function representations for the process and the controller. Include the external inputs r and d, and label the signals u and y.

(d) In the closed-loop, what are the transfer functions from R to Y and from D to Y.

(e) In the closed-loop, what are the transfer functions from R to U and from D to U.

(f) For what values of K_P and K_I is the closed-loop system stable?

(g) Choose K_P and K_I so that the closed-loop characteristic equation has roots with $\xi = 0.707 (= \frac{1}{\sqrt{2}})$ and $\omega_n = 2$.