Math 54, Section 1: Differential equations and linear algebra.

Fall 1994, H. W. Lenstra, Jr.

Midterm, October 31, 1994.

Name:

Section number:

T. A.:

List of discussion sections:

101 S. Simic
102 A. Gottlieb
103 G. Anderson
104 G. Anderson
105 S. Simic
106 T. Walker
107 A. Gottlieb
108 L. Pyle
109 L. Pyle

1	
2	
3	
4	
Total	

Problem 1. *(25 points)*

Let W be the set of 3×3-matrices A for which $A^T = -A$.

(a) Show that W is a subspace of the vector space of all 3×3-matrices.

(b) Find a basis for W. What is the dimension of W? Why?

Solution:

Problem 2. *(25 points)*
Consider the 3×3-matrix

$$A = \begin{pmatrix} a & 0 & 1 \\ 0 & b-1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) For which pairs of numbers a, b does A have rank 3? Explain your answer.

(b) For which pairs of numbers a, b does A have rank 2? Explain your answer.

Solution:

Problem 3. *(25 points)*
Let $C[0,1]$ be the vector space of all continuous real-valued functions on the interval $[0,1]$, provided with the usual inner product

$$\langle \mathbf{f}, \mathbf{g} \rangle = \int_0^1 f(x)g(x)dx.$$

Find a non-zero function in $C[0,1]$ that is orthogonal to the function x. Explain your method.

Solution:

Problem 4. *(25 points)*

Do two of (a), (b), *and* (c). *Cross out the one you don't want to be graded.*

Let A be a 3×3-matrix for which the vectors

$$\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

are eigenvectors, with respective eigenvalues $2, -1, 0$.

(a) Compute the matrix A.

(b) Compute

$$A^{1995} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

(c) Is A invertible? Justify your answer.

Solution: