Spring 2001

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MATH 54 MIDTERM (yellow)

Do not write your answers on this sheet. Instead please write your name, your student ID, your TA's name, your section time, "yellow," and all your answers in your blue books. In general, you must show your work to get credit. Total: 100 pts., 50 minutes.

(1) (10 pts. each) For each of (a)-(c) below: If the statement is always true, write TRUE and briefly explain why. If the statement is sometimes false, write FALSE and give a numerical counterexample. For example, if the problem were

"If A is a square matrix and $A^2 = \mathbf{0}$, then $A = \mathbf{0}$."

you would write

"FALSE. Take $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. Then $A^2 = \mathbf{0}$, but $A \neq \mathbf{0}$."

(a) If A and B are invertible $n \times n$ matrices, then $A^T + B$ is invertible.

(b) If the rows of an $m \times n$ matrix A are linearly independent, then the columns of A are linearly independent.

(c) If A is an $m \times n$ matrix with $m < n$, and $\mathbf{b} \in \mathbb{R}^m$, then the linear system $A\mathbf{x} = \mathbf{b}$ has at least one solution $\mathbf{x} \in \mathbb{R}^n$.

(2) (10 pts. each) Let P_3 denote the vector space of polynomials of degree less than or equal to 3, including the zero polynomial. Let S be the subset

$$\{1 + x + 3x^2, 4 + 7x + 3x^3, 2 + 3x + 2x^2 + x^3\}$$

of P_3, and let $V = \text{Span}(S)$.

(a) Is S linearly dependent? (Explain how you are solving this problem.)

(b) Find a subset T of S that is a basis for V. (Give T, and explain how you know it is a basis for V.)

(3) (15 pts.) Let V denote the vector space of infinitely differentiable functions $f : \mathbb{R} \to \mathbb{R}$. Is there a linear transformation $T : V \to V$ such that $\text{NS}(T) = \text{Span}\{e^{2x}\}$? (If YES, describe one such T, but you do not need to prove that T is a linear transformation or that $\text{NS}(T) = \text{Span}\{e^{2x}\}$. If NO, just say NO.)

(4) (15 pts.) Let V be the vector space of continuous functions $f : \mathbb{R} \to \mathbb{R}$. Are the functions $\sin x$, $\sin(x + 1)$, $\sin(x + 2)$ linearly dependent in V? Explain.

(5) (20 pts.) Is there a square matrix A such that $\text{CS}(A) = \text{NS}(A)$? (If YES, give A and show that $\text{CS}(A) = \text{NS}(A)$. If NO, explain briefly why no such A exists.)

This is the end! At this point, you may want to look over this sheet to make sure you have not omitted any problems (especially the ones with parts).