**Math 54, Section 1: Differential equations and linear algebra.**

Fall 1994, H. W. Lenstra, Jr.

Midterm, September 26, 1994.

**Name:**

**Section number:**

**T. A.:**

List of discussion sections:

101 S. Simic

102 A. Gottlieb

103 G. Anderson

104 G. Anderson

105 S. Simic

106 T. Walker

107 A. Gottlieb

108 L. Pyle

109 L. Pyle

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|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| Total | |

**Problem 1.** *(25 points)*
Let the matrix $A$ be defined by

$$A = \begin{pmatrix} 7 & -3 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 7 & -3 & 0 \\ 0 & 0 & 0 & 1 & -3 \\ -3 & 0 & 0 & 0 & 7 \end{pmatrix}.$$

(a) Calculate the determinant of $A$.

(b) Calculate the determinant of $A^3$ without computing $A^3$.

**Solution:**

**Problem 2.** *(25 points)*
Consider the system of linear equations

$$\begin{aligned} x + 2y + az &= 0, \\ -x \qquad + \ z &= 0, \\ ax - \ y + \ z &= 0. \end{aligned}$$

Find the values of $a$ for which the system has a unique solution; infinitely many solutions; no solution.

**Solution:**

**Problem 3.** *(25 points)*
Let $T: R^2 \to R^2$ be the linear operator defined by $T(x, y) = (7x, 4x + 3y)$. Determine the eigenvalues of $T$, and find for every eigenvalue an eigenvector.

**Solution:**

**Problem 4.** *(25 points)*

(a) Let $A$ and $B$ be square matrices. Suppose that $A$ is invertible and that $BAB = A$. Show that $B$ is invertible.

(b) Let $C = (c_{ij})$ be a $2 \times 2$ matrix satisfying

$$c_{11} = \tfrac{4}{5}, \quad c_{21} = \tfrac{3}{5}, \quad C^T C = I, \quad \det C > 0.$$

Determine $C$.

**Solution:**