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Fall 2002, Math 250A

Midterm

16 October, 2002
1:10-2:00 PM

1. (48 points; 12 points each.) For each of the items listed below, either *give an example,* or give a brief reason why *no example exists.* (If you give an example, you do *not* have to prove that it has the property stated. Examples should be specific for full credit; i.e., even if there are many objects of a given sort, you should name one.)

(a) A group G, and two nonisomorphic finite G-sets with the same number of elements.

(b) A surjective group homomorphism from the alternating group A_6 to the alternating group A_5.

(c) Groups G and H and a homomorphism $f\colon G \to H$ which is one-to-one, but such that there does not exist a homomorphism $g : H \to G$ for which gf is the identity endomorphism of G.

(d) A finite group G, a prime p, and two distinct p-Sylow subgroups of G.

2. (18 points) Prove: If p and q are primes such that $q \equiv 1 \pmod{p}$, then there exists a noncommutative group of order pq. (This was proved in the Companion. In proving it here, you may assume any results proved in Lang, and any proved in the Companion up to the point where that result was obtained.)

3. (16 points) Let I and J be ideals of a commutative ring A. Show that if the ideal $I \cap J$ is prime, then either $I \subseteq J$ or $J \subseteq I$.

4. (18 points) Show that if $f\colon G \to F$ is a surjective group homomorphism, where F is a free group and G an arbitrary group, then there exists a group homomorphism g : $F \to G$ such that fg is the identity endomorphism of F.