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Spring 2000, Math 113, Section 3

7 April, 2000

70 Evans Hall

Second Midterm

1:10-2:00 PM

1. (36 points, 6 points apiece) Find the following. Correct answers will get full credit whether or not work is shown.

(a) The characteristic of the ring $\mathbb{Z}_6[x]$.

(b) The remainder when 4^{102} is divided by 11.

(c) The order of a 5-Sylow subgroup of $S_8 \times S_{10}$.

(d) The set of solutions in \mathbb{Z} of the congruence $3x \equiv 2 \pmod{120}$.

(e) The order of the factor group S_4/A_4 .

(f) A factorization of the polynomial $x^2 - 2 \in \mathbb{Z}_7[x]$ into irreducible polynomials.

2. (34 points) Let G be a group, N a normal subgroup of G , and m a positive integer. Prove that the following two conditions are equivalent (i.e., that the first holds if and only if the second holds):

(i) Every element x of the factor group G/N satisfies $x^m = e_{G/N}$.

(ii) For every $g \in G$, one has $g^m \in N$.

3. (30 points; 6 points each.) For each of the items listed below, either give an *example*, or give a brief reason why *no example exists*. (If you give an example, you do not have to prove that it has the property asked for.)

(a) A nonabelian simple group.

(b) A polynomial in $\mathbb{Q}[x]$ that is reducible over \mathbb{Q} , but has no roots in \mathbb{Q} .

(c) A factorization of $x^6 + 5x^3 + 25x + 60 \in \mathbb{Z}[x]$ into polynomials of smaller degree.

(d) A transitive S_3 -set X . (If you give an example, be sure to indicate the action of S_3 on X .)

(e) A group containing an element of order 2000, but no element of order 2.