George M. Bergman
70 Evans Hall

Spring 2000, Math 113, Section 3

First Midterm

28 February, 2000
1:10-2:00 PM

1. (40 points, 8 points apiece) Find the following. Correct answers will get full credit whether or not work is shown.

(a) The integers q and r such that $0 \leq r < 4$ satisfying $13 = 4q + r$.

(b) The inverse of 3 in $<\mathbb{Z}_4, +>$.

(c) The left coset in S_3 of the subgroup $\{\iota, (1,2)\}$ which contains the element $(1,3)$.

(d) The kernel of the homomorphism $\varphi: \mathbb{Z} \to S_5$ given by $\varphi(n) = (1,2,4)^n$.

(e) An expression for the element $(1,2,3)(4,5) \in S_5$ as a product of transpositions.

2. (20 points) Show that for any three elements a, b, c of a group G, the equation $axb = c$ has a unique solution in G, i.e., that there is a unique element $x \in G$ satisfying that equation. (You may either use results proved in the text, or give a self-contained proof. The two methods are about equally easy.)

3. (40 points; 10 points each.) For each of the items listed below, either give *an example,* or give a brief reason why *no example exists.* (If you give an example, you do not have to prove that it has the property asked for.)

(a) Two groups G and H and a homomorphism $\varphi: G \to H$ whose kernel is neither $\{e\}$ nor G.

(b) An element of order 10 in the group $S_3 \times \mathbb{Z}_{15}$.

(c) A finite group which is not cyclic.

(d) A group G with two cyclic subgroups H and K whose intersection, $H \cap K$, is not cyclic.