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Spring 1999, Math 113, Section 2

Second Midterm

9 April, 1999
1:10–2:00 PM

1. (40 points, 8 points apiece) Find the following. Correct answers will get full credit whether or not work is shown.

(a) The kernel of the homomorphism $\varphi: \mathbf{Z} \to \mathbf{C}^{\times}$ given by $\varphi(n) = i^n$.

(b) The stabilizer subgroup G_x, where $G = S_3$, acting on the set $\{1,2,3\}$ by $\sigma i = \sigma(i)$, and $x = 2$.

(c) The orbit Gx, where G and x are as in the preceding part.

(d) $|\mathrm{Inn}(D_4)|$, i.e., the order of the group of inner automorphisms of the group of rigid motions of a square.

(e) The quotient and remainder on dividing x^4 by $2x+1$ as polynomials over the field $\mathbf{Z}_5$. Both should be written with coefficients taken from the set $\{0, \ldots, 4\}$.

2. (30 points) Describe an infinite set of integers n such that the polynomial $x^5 + 100x^2 + n$ over the integers is irreducible, and an infinite set of integers n for which that polynomial is reducible. Briefly justify your claims of irreducibility and reducibility. If in either case you cannot give an infinite set, you may get partial credit by giving a finite set. (Hint for one of the above parts: can you choose n so that the polynomial has a root?)

3. (a) (20 points) Show that if $\varphi: G \to H$ is a homomorphism, with G and H finite groups, and n is a positive integer such that $\varphi(G)$ contains an element of order n, then G also contains an element of order n.

(b) (10 points) Show by example that if G and H are not required to be finite, the above statement is false.