MATH 110 – EXAM 2

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Time limit: 50 minutes

Name:

SID:

Please put away all books, papers, calculators, pagers, and other electronic devices. Please write your name on each page of the exam. Don't trust staples to hold the exam together.

Please write all answers in **complete sentences**. You will not receive full credit if your answers are not in complete sentences. The final answer of a computation should also be written as a complete sentence.

There are two extra blank pages at the end of the exam. You may use these for computations, but **I will not read them.** Please transfer all final answers to the page on which the question is posed.

Problem:	Your score:	Total points
1		30 points
2		20 points
3		20 points
4		30 points
Total:		100 points

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1. **True/False.** Please label each of the following statements as *True* or *False*. Please explain your answer: give a brief justification or a counterexample.

   (a) Let $A \cdot x = b$ denote a system of n equations in n variables, so $A \in M_{n\times n}(\mathbb{F})$. Then there is always a solution to this system of equations.

   (b) If P is a permutation matrix, then $\det(P) = 1$ or $\det(P) = -1$.

(Continued on the next page)

(c) Let $T : V \to V$ be a linear transformation, and let β and γ be two ordered bases of T. Then $\det[T]_\beta^\beta = \det[T]_\gamma^\gamma$.

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2. Let $\begin{bmatrix} x \\ y \end{bmatrix}$ be the standard coördinates on $\mathbb{R}^2$. Let ℓ be the line $y = m \cdot x$ in $\mathbb{R}^2$ for some $m \neq 0$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by reflection across the line ℓ. For an example, see the figure below. Write down an expression for $T(x, y)$ (in terms of the standard coördinates).

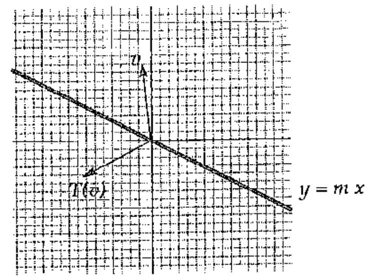

FIGURE 1. This shows a line $y = m \cdot x$, a vector v, and the vector $T(v)$ reflected across the line.

3. Let $A, B \in M_{n \times n}(\mathbb{F})$. Suppose there is a non-zero vector $v \in \mathbb{F}^n$ such that $A \cdot v = B \cdot v$. Show that $\det(A - B) = 0$.

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4. Suppose that A is a 3×4 matrix and its reduced row echelon form is R:

$$R = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Let L_A denote the linear transformation corresponding to left multiplication by A.

(a) What are the dimensions of $R(L_A)$ and $N(L_A)$?

(b) Find a matrix B with *no zero entries* (if possible) whose reduced row echelon form is this same R.

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(c) Consider the system of linear equations

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

For what values $a, b, c \in \mathbb{R}$ does this system have a solution? Suppose a, b, and c are values such that there is a solution. In this case, what are *all* the solutions, in terms of a, b, and c?