MATH 110 – EXAM 1

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Time limit: 50 minutes

Name:

SID:

You may not consult any books or papers. You may not use a calculator or any other computing or graphing device other than your own head!

Please write your name on *EVERY* page of this exam. (This guarantees 4 points if you do it, and you lose 4 points if you do not!)

There are two extra problems at the end of the exam. They *are not required*, and you will not receive extra credit for them. They are meant for your amusement.

There are two extra blank pages at the end of the exam. You may use these for computations, but **I will not read them.** Please transfer all final answers to the page on which the question is posed.

Problem:	Your score:	Total points
1		25 points
2		25 points
3		25 points
4		25 points
Total:		100 points

1. **Short answer.** Please answer the following questions.

   Please note that for the True/False questions, you are required to justify your answer: give a proof or counterexample.

   If there is a computation, please circle your final answer. Let A^{T} denote the transpose of A, and $tr(A)$ the trace of A.

   (a) Let V be a vector space over a field $\mathbb{F}$, and $S \subseteq V$ a subset of V. What do you need to prove in order to show that S is a basis of V?

   (b) **True or False:** Suppose V is a vector space. Let U and W be subspaces of V. Then $U \cup W$ is also a subspace of V.

   (c) **True or False:** Consider the map
   $$T : M_{2\times 3}(\mathbb{F}) \to M_{3\times 2}(\mathbb{F})$$
   that sends a matrix A to its transpose $T(A) = A^{\mathrm{T}}$. This is a linear transformation.

2. **Bases.** The set
$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix} \right\}$$
   is a basis for $M_{2\times 2}(\mathbb{R})$ as a vector space over $\mathbb{R}$ (you *do not* need to prove this).

3. **Subspaces.** Let $\mathbb{F}$ be a field, and consider the vector space $M_{n\times n}(\mathbb{F})$ over $\mathbb{F}$. For every scalar $a \in \mathbb{F}$, define
$$M_a = \left\{ A \in M_{n\times n}(\mathbb{F}) \;\middle|\; tr(A) = a \right\}.$$

4. **Dimension.** Let W_1 and W_2 be subspaces of a finite-dimensional vector space V. Recall that the sum of W_1 and W_2 is
$$W_1 + W_2 = \left\{ u + v \;\middle|\; u \in W_1 \text{ and } v \in W_2 \right\}.$$

   Prove that
$$\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2).$$