Midterm #1

Math 121A (Section 2) - Fall 2001 Tokman

Each problem counts 10 points

Problem # 1. Find all solutions of the equation

$$z^3 = -8i,$$

write them in a polar ($|z|e^{i\arg(z)}$) and rectangular ($\mathrm{Re}(z) + i\mathrm{Im}(z)$) forms and plot them in the complex plane.

Problem # 2. Find the sum of the series

$$(\pi/6 + i) - \frac{(\pi/6+i)^3}{3!} + \frac{(\pi/6+i)^5}{5!} + \ldots = \sum_{n=0}^{\infty}(-1)^n \frac{(\pi/6+i)^{2n+1}}{(2n+1)!}$$

and write it in a rectangular form $\mathrm{Re}(z) + i\mathrm{Im}(z)$.

Problem # 3. Find the first three terms of the two-variable Maclaurin series for

$$\frac{\sinh(xy)}{1+x^2y}.$$

Write down both the derivation and the final answer!

Problem # 4. (a) Derive the formula

$$\tanh^{-1}(z) = \frac{1}{2}\ln\frac{1+z}{1-z}$$

from the definition of $\tanh z$ (or definitions of $\sinh z$ and $\cosh z$).

(b) Use the derived formula to compute

$$(i)^{2\tanh^{-1}(i)}.$$

Problem # 5. Find the interval of convergence of the following series (including end points tests!):

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}(-3)^n}.$$

Justify your answer by mentioning the theorems/tests that you use to draw conclusions about convergence, and state explicitly if the convergence is absolute or conditional.

Problem # 6. Compute dz/dt given

$$z = x^y,$$

$$x = \sin t,$$

$$y = \tan t.$$

Problem # 7. Does the following series converge?

$$\sum_{n=1}^{\infty} \frac{n^3 - \ln n}{2^n + 10n}$$

Justify your answer by mentioning the theorems/tests that you use to draw conclusions about convergence.