Math 121A Final, 2001 December 14 8:00-11:00. Borcherds

Please make sure that your name is on everything you hand in.
You are allowed calculators and 1 page of notes.
Answer as many questions as you can.
All questions have about the same number of marks.

1. Evaluate $\sin(\theta) + \sin(2\theta) + \cdots + \sin(n\theta)$.

2. Evaluate the integral
$$\int_0^\infty \frac{\sqrt{x}dx}{(1+x)^2}.$$

3. Expand the function $f(x)$ in a sine-cosine Fourier series, where $f(x)$ is 1 if $0 \le x < \pi$, 0 if $-\pi \le x < 0$, and $f(x+2\pi) = f(x)$.

4. Calculate the Laplace transform $\int_0^\infty e^{-pt} f(t)dt$ of $f(t) = e^{-at}\sin(bt)$.

5. Use Laplace transforms to solve the differential equation $y'' - 4y' + 4y = 4$, $y(0) = 0$, $y'(0) = -2$. (If y has Laplace transform Y then y' has Laplace transform $pY - y(0)$ and y'' has Laplace transform $p^2Y - py(0) - y'(0)$. Also 1 has Laplace transform $1/p$ and e^{-at} has Laplace transform $1/(p+a)$.)

6. Find the exponential Fourier transform
$$g(\alpha) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-i\alpha x}dx$$
for the function $f(x)$ defined by $f(x) = x$ if $|x| < 1$, $f(x) = 0$ if $|x| \ge 1$.

7. Write and solve the Euler equations $(d/dx)(\partial F/\partial y') = \partial F/\partial y$ to make the following integral stationary:
$$\int_{x_1}^{x_2} (y'^2 + \sqrt{y})dx$$

8. Change the dependent variable from y to x in the following integral, then write and solve the Euler equation to make it stationary.

$$\int_{x_1}^{x_2} (y'^2 + y^2)dx$$

9. Calculate the inverse Laplace transform

$$f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(z)e^{zt}dz$$

when F is the function $F(z) = 1/(z^4 - 1)$.

10. Find the shortest distance from the origin to the surface

$$3x^2 + y^2 - 4xz = 4.$$