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Fall 1999, Math 1B

Second Midterm – makeup exam

2 November, 1999
8:10-9:30 AM

1. (36 points, 6 points apiece) Find the following. If an expression is undefined, say so.

(a) $\Sigma_{n=2}^{\infty} 5^n/n^5$.

(b) $\Sigma_{n=1}^{\infty} (2^{-n} + 2^{-n/2})$.

(c) The set of all real numbers p such that $\Sigma_{n=2}^{\infty} n^{-p}(\ln n)^{-2}$ converges.

(d) The Maclaurin series for $\sin \pi x$.

(e) The Taylor series for $1/x^3$ centered at $x = -1$.

(f) The solution to the differential equation $xy' = y^2+1$ satisfying the initial condition $y(1) = 1$.

2. (16 points) Let a and b be real numbers. Prove that $\Sigma_{n=1}^{\infty} \frac{1}{n^a + n^b}$ converges if and only if at least one of a and b is >1.

3. (30 points, 6 points apiece) For each of the items listed below, give either *an example of the situation described*, or a brief reason why *no such example exists*. (If you give an example, you are *not* asked to show that it has the asserted property.)

(a) A power series $\Sigma_{n=0}^{\infty} a_n (x-1)^n$ which converges only at $x = 2$.

(b) A power series $\Sigma_{n=1}^{\infty} a_n x^n$ which converges for all $x \in [-1, 1]$ and no other x.

(c) A power series $\Sigma_{n=0}^{\infty} a_n (x+2)^n$ which converges for all real numbers x.

(d) A series $\Sigma_{n=1}^{\infty} a_n$ which converges, but such that $\Sigma_{n=1}^{\infty} |a_n|$ diverges.

(e) A series $\Sigma_{n=1}^{\infty} a_n$ which diverges, but such that $\Sigma_{n=1}^{\infty} |a_n|$ converges.

4. (18 points) (a) (7 points) Find the first three terms (i.e., the constant, linear, and square terms) of the Taylor series for e^{-x} centered at $x=2$.

(b) (11 points) Prove using the formula for the remainder (''Taylor's Formula'') that for all x in the interval $[1.5, 2.5]$, the sum of the above three terms approximates e^{-x} to within $e^{-3/2}/48$.