MATH 1B FINAL

December 19, 2002 12:30–3:30 PM

H. Wu

Your Name: ____________________________

Your GSI's Name: ____________________________

Instructions

(1) Check that you have all **10** pages of this exam booklet.

(2) **Be sure to show all your steps.**

(3) You may not use any fact that has not been covered in the course to do the exam.

EXAM SCORES					
Problem	Max	Your Score	Problem	Max	Your Score
1	20		6	20	
2	20		7	20	
3	15		8	20	
4	15		9	40	
5	25		10	5	
TOTAL			/200		

1. (20 pts.) Solve: $y' + 2xy = 2x^3, \quad y(0) = 1.$

2 (20 pts.) (a) Solve: $y'' - 2y' + 10y = 0, \quad y(0) = 1,$ and $\quad y(\frac{\pi}{2}) = 2.$

MORE SPACE FOR PART (a) ON THE NEXT PAGE

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(b) Solve: $y'' - 8y' + 16y = 0$, $y(0) = 4$ and $y'(0) = 0$.

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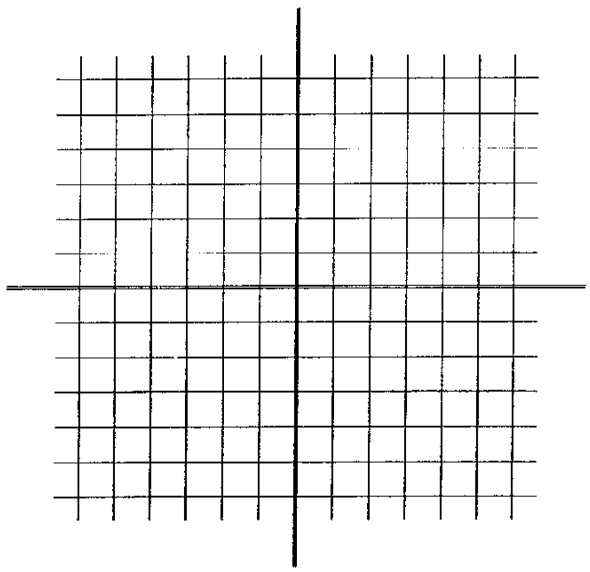
3 (15 pts.) Sketch the direction field for the equation $y' = x^2 + y^2$ and sketch the graph of the solution curve $y(x)$ which satisfies $y(0) = 1$.



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4 (15 pts.) Evaluate $\int_0^1 \sin(x^2)\, dx$ as an infinite series, and show directly that the series is convergent.

5 (25 pts.) In an idealized model, the trout population in a pond at time t, $P(t)$, is assumed to satisfy the equation

$$(*) \qquad \frac{dP}{dt} = P - \frac{1}{10^3}P^2$$

(a) If at the beginning ($t = 0$), the trout population is $P(0) = 10^2$, solve explicitly for $P(t)$. Show all your steps. (b) What is the value of $P(t)$ as $t \to \infty$? (c) Suppose only 10 trouts are left in the pond after a fishing season and suppose equation $(*)$ continues to hold for the trout population. What will the trout population approximately be after a long, long time if it is left alone?

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6 (20 pts.) Solve: $y'' - 3y' + 2y = e^{2x}$, $y(0) = -1$ and $y'(0) = 0$.

7 (20 pts.) (a) Solve: $y'' - xy' - 2y = 0$, $y(0) = 0$ and $y'(0) = 5$. (b) Write the solution in terms of elementary functions.

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8 (20 pts.) Use the *method* in the proof of the Integral Test to give a direct explanation of why $\sum_{n=3}^{\infty} \frac{1}{n \ln n}$ is divergent, but *do not* make use of the Integral Test itself.

9

9 (40 pts.) Write **T** (true) or **F** (false) next to each of the following statements. You get +4 points for each correct answer, 0 point for having the self-control not to put down an answer, and −2 for each wrong answer.

(a) If $\sum_0^\infty a_n$ is convergent, then the radius of convergence of the power series $\sum_0^\infty a_n x^n$ is at least 1.
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(b) It is known that the equation $4x'' + 16x = 5\sin 2t$ describes the motion of a spring which has a mass of 4 kg attached to it, whose spring constant is 16, and which is subject to an external force of magnitude $5\sin 2t$. Then the motion of the spring is *periodic* for all t. ____________

(c) Suppose $\sum_0^\infty a_n$ is convergent and $a_n > 0$ for all n, then $\sum_0^\infty \frac{1}{\sqrt{a_n}}$ is divergent. ____________

(d) Suppose all the numbers $a_1,\ a_2,\ a_3,\ \ldots$ are negative and suppose $(a_1 + a_2 + a_3 + \cdots + a_n) \geq -11$ for all $n = 0, 1, 2, \ldots$. Then $\sum_0^\infty a_n$ is convergent. ____________

(e) A particular solution of $y'' - 2y' + 5y = 2e^x \cos 2x$ is of the form $Ae^x \cos 2x + Be^x \sin 2x$ for some constants A and B. ____________

(f) $\int \sec x\, dx = \ln|\sec x + \tan x| + c$, where c is a constant. ____________

(g) It is known that the arc length of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$ is $\frac{\sqrt{5}}{2} + \frac{\ln(\sqrt{5}+2)}{4}$. The arc length of the parabola $y = 4x^2$ from $(0,0)$ to $(1,4)$ is then $2\sqrt{5} + \ln(\sqrt{5}+2)$. ____________

(h) We can use the root test to give a proof of the convergence of the geometric series $\sum_0^\infty r^n$ for any r satisfying $|r| < 1$. ____________

(i) The solution $y(x)$ of the equation $y'' + y^2 = -5$ so that $y(0) = 1$ and $y'(0) = 0$ must satisfy $y(x) \leq 1$ for all x. ____________

(j) If $0 \leq f(x) \leq \frac{1}{x^{\sqrt{2}}}$ for $1 \leq x < \infty$, then $\int_1^\infty f(x)dx$ is convergent. ____________

10 (5 pts.) Who is Bill Walsh? (5 points for an answer, and −5 for no answer).