University of California, Berkeley
Mechanical Engineering
ME106 Fluid Mechanics
2nd Test, S06 *Prof S. Morris*

NAME SOLUTIONS

1. (40) Find the difference $T_0 - T_\infty$ between the stagnation and atmospheric temperatures for an aircraft moving at speed $V = 200$ m/s. The specific heat $c_p = 1$kJ/kg·K.

BE along stagnation streamline from ∞ to O:

$$\underbrace{\frac{1}{2}V^2 + c_p T_\infty}_{\text{at } \infty} = \underbrace{c_p T_0}_{\text{at } O}$$

−10: Start from a different (correct) equation, but mess up algebra.

−2: sign

$$\Rightarrow \quad \boxed{T_0 - T_\infty = \frac{V^2}{2c_p}} \quad +35$$

$V = 200$ m/s
$c_p = 10^3$

$$\Rightarrow T_0 - T_\infty = \frac{(200)^2}{2000} = \frac{40000}{2000} = 20\text{ K}.$$

$$\boxed{T_0 - T_\infty = 20\text{ K}} \quad +5$$

−2 (→ factor 2, typically) if incorrect for arithmetic

−5 if incorrect for confusion of kJ with J (→ factor $\underline{10^3}$)

This error is independent of use of calculator!!

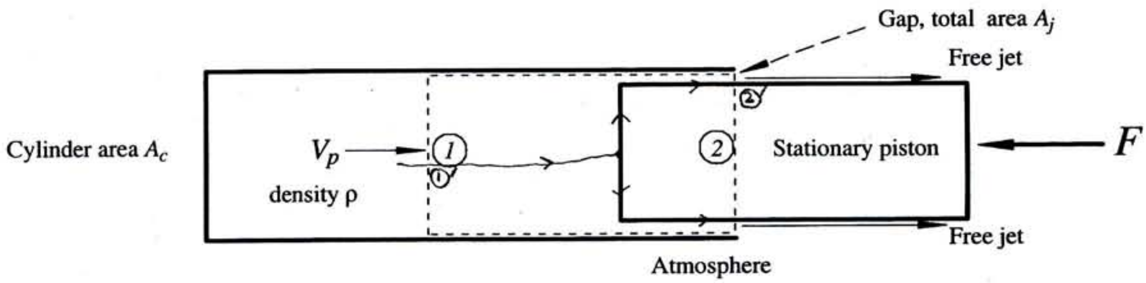
−2 units

PLEASE PRINT YOUR NAME ON THIS PAGE

CROSS OUT (ALL) BLANK PAGES WHEN GRADING

** REMIND THEM THAT p_a ACTS ON THE OUTSIDE OF THE PISTON IN ADDITION TO F **

2. **(70)** In a hydraulic buffer, the force F applied to the buffer piston is balanced by the pressure force exerted on the piston by the fluid. In the figure, the axes are taken to be fixed in the piston. Fluid moves towards the stationary piston with speed V_p, and then leaves the cylinder as a free jet. The flow is quasi–steady, incompressible, and effectively inviscid.



(a) By using the incompressible form of the Bernoulli equation along a clearly identified streamline, find the pressure p_1 acting at face 1 of the control volume in terms of atmospheric pressure p_a, ρ, V_p and the unknown velocity V_j in the free jet.

(b) By balancing mass and momentum on the contents of the control volume shown in the figure, and by using the result from part (a), find F in terms of ρ, A_c, V_p and A_j.

(a) By apply BE along stagnation streamline from point 1' to 2'

$$p_1 + \tfrac{1}{2}\rho V_p^2 = p_{2'} + \tfrac{1}{2}\rho V_j^2 \qquad (*)$$

$$p_{2'} = p_a \quad \text{free jet}$$

$$\Rightarrow \boxed{p_1 = p_a + \tfrac{1}{2}\rho (V_j^2 - V_p^2) \qquad (1)}$$

15

−5 if BE stated correctly at (*), but (1) is wrong, eg with sign errors

PAPER 52 CONSISTENTLY REPLACED A_c BY $A_c - A_j$, BUT OTHERWISE WAS COMPETENT. I DEDUCTED 10 ONLY, THOUGH THE ERROR AFFECT BOTH THIS MASS & MOM. BALANCE

(b) MASS BALANCE $\qquad V_p A_c = V_j A_j \qquad (2)$ $\qquad$ 10

MOM. BALANCE $\qquad \underbrace{\rho V_p A_c}_{\dot m}(V_j - V_p)$ = resultant force to right

40

$= p_1 A_c - F - p_a A_c$

+20

(p_a acts on jet and on piston back)

−10 if all three forces present, but with errors.

−20 if ANY OF 3 FORCES IS MISSING

Deduct 5 for sign errors momentum flow rates

10

$= \underbrace{(p_1 - p_a) A_c - F}_{+20} \qquad (3)$

−10 Here if area missing, or INCORRECT & dimensionally incorrect.

Deduct here (if) dimensions are wrong & (if) final result for F is wrong (so the mistake was not subsequently corrected)

NOTE IF $F' = F + p_a (A_c - A_j)$

RHS = $p_1 A_c - p_a A_j - F'$ which is an acceptable form (even though it is awkward)

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Eliminate $p_1 - p_a$ between ①, ③

$$\Rightarrow \quad \rho V_p A_c (V_j - V_p) = \tfrac{1}{2}\rho (V_j^2 - V_p^2) A_c - F$$

$$\therefore \quad F = \tfrac{1}{2}\rho A_c (V_j^2 - V_p^2) - \rho V_p A_c (V_j - V_p)$$

$$\frac{F}{\frac{1}{2}\rho A_c V_p^2} = \left(\frac{V_j}{V_p}\right)^2 - 1 - 2\left(\frac{V_j}{V_p} - 1\right)$$

$$= \left(\frac{V_j}{V_p}\right)^2 - 2\left(\frac{V_j}{V_p}\right) + 1$$

$$= \left(\frac{V_j}{V_p} - 1\right)^2$$

By ② $\quad \frac{V_j}{V_p} = \frac{A_c}{A_j}$

+5 Final result

$$\Rightarrow \boxed{\frac{F}{\frac{1}{2}\rho A_c V_p^2} = \left(\frac{A_c}{A_j} - 1\right)^2} \quad ⊛$$

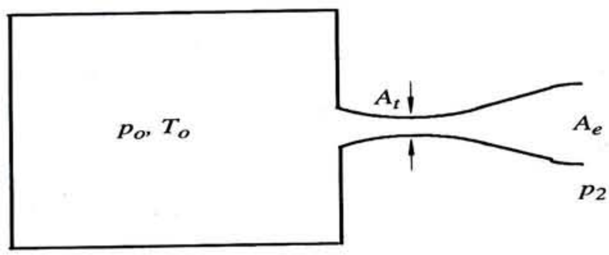
* Give full credit if eq. ①-③ are present & correct AND if result is given in form equivalent to ⊛, provided answer given in terms of variables stated in question

NOTE

$$\frac{F}{\frac{1}{2}\rho A_c V_p^2} = \left(\frac{A_c}{A_j} - 1\right)^2 + p_a (A_c - A_j)$$

is acceptable as final answer (amounts to different definition of F.)

3. (90) As shown in the figure, an ideal gas flows isentropically from a large reservoir through a converging–diverging nozzle having **fixed** exit area A_e. Atmospheric pressure p_2 is only slightly below the stagnation pressure p_0, so that the flow at the exit is subsonic, but compressible. The flow is isentropic, and one–dimensional.



(a) Assuming that the flow is subsonic within the throat, find the relation giving the mass flow rate $\dot{m}$ in terms of the specific heat ratio γ, stagnation sound speed c_0, stagnation density ρ_0, pressure ratio p_2/p_0, and A_e.

(b) Find the relation giving the ratio of the pressure p_t within the throat to the stagnation pressure p_0 as an implicit function of $\dot{m}$, γ, ρ_0, c_0 and the throat area A_t.

(c) What is the smallest value of p_t/p_0 attainable by reducing A_t with $\dot{m}$ fixed?

(d) Find $(A_t/A_e)^2$ as a function of p_2/p_0 and γ for the case identified in part (c).

(e) What happens to $\dot{m}$ if A_t is reduced, with p_0 and p_2 fixed, so that A_t/A_e is made less than the value derived in part (d)? Your answer must include the simplest equation that could be used to calculate $\dot{m}$ correctly in this case.

(a) Because the flow is subsonic in the throat it is subsonic and isentropic everywhere.

By the compressible form of Bernoulli's equation,

$$V^2 = 2c_pT_0\left\{1-\left(\frac{p}{p_0}\right)^{1-\frac{1}{\gamma}}\right\}$$

+15 (*)

$c_p = \frac{\gamma}{\gamma-1}R$, $c_0^2 = \gamma R T_0$ $\Rightarrow$ $$\boxed{V^2 = \frac{2}{\gamma-1}c_0^2\left\{1-\left(\frac{p}{p_0}\right)^{1-\frac{1}{\gamma}}\right\}} \quad (1)$$

$\boxed{\text{At exit } p=p_2}$ $\Rightarrow$ $$V_e^2 = \frac{2}{\gamma-1}c_0^2\left\{1-\left(\frac{p}{p_0}\right)^{1-\frac{1}{\gamma}}\right\}$$

$$\rho_e = \rho_0\left(\frac{p}{p_0}\right)^{1/\gamma} \quad +5 \quad (**)$$

+20 $\Rightarrow$ $$\boxed{\dot{m}^2 = \frac{2}{\gamma-1}\rho_0^2c_0^2\left\{\left(\frac{p_2}{p_0}\right)^{2/\gamma} - \left(\frac{p_2}{p_0}\right)^{1+\frac{1}{\gamma}}\right\}A_e^2} \quad (2)$$

$\dot{m}$ as fn of γ, c_0, ρ_0, p_2/p_0, A_e

GIVE FULL CREDIT HERE IF BOTH (*) & (**) ARE PRESENT, EVEN IF FINAL RESULT (2) NOT CORRECTLY GIVEN

* || GIVE FULL CREDIT FOR (b) IF RESULT CONSISTENT WITH ~~[illegible]~~ AS (a) WITH $p_2 \to p_t$

(b) Because the flow is subsonic at the throat, ② holds there with $(p_2, A_e) \to (p_t, A_t)$

(+10)

$$\boxed{\dot{m}^2 = \frac{2}{\gamma-1}\rho_0^2 c_0^2 \left\{ \left(\frac{p_t}{p_0}\right)^{2/\gamma} - \left(\frac{p_t}{p_0}\right)^{1+\frac{1}{\gamma}} \right\} A_t^2} \quad ③$$

p_t/p_0 as implicit function of A_t, $\dot{m}$, γ, ρ_0, c_0

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FINAL RESULT MUST BE GIVEN CORRECTLY FOR CREDIT.

(+20)

(c) $p_t \geq p_*$ where $\frac{p_*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

Full credit if this is present

(d) For $p_t = p_*$, ③ $\Rightarrow$ $\dot{m}^2 = \frac{2}{\gamma-1}\rho_0^2 c_0^2 \left\{ \left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \right\} A_t^2$

$$= \frac{2}{\gamma-1}\rho_0^2 c_0^2 \left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} \frac{\gamma-1}{\gamma+1} A_t^2$$

(+20)

$$\boxed{\dot{m}^2 = \underbrace{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \rho_0^2 c_0^2}\, A_t^2} \quad ④ \qquad \text{, (i.e. } \dot{m}^2 = \rho_*^2 c_*^2 A_t^2 \text{)}$$

Eliminate $\dot{m}^2$ between ②, ④:

$$\Rightarrow \boxed{\left(\frac{A_t}{A_e}\right)^2 = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{2}{\gamma-1}\left\{ \left(\frac{p_2}{p_0}\right)^{2/\gamma} - \left(\frac{p_2}{p_0}\right)^{1+\frac{1}{\gamma}} \right\}} \quad ⑤$$

$(A_t/A_e)^2$ as a function of γ, p_2/p_0

(e) If you reduce A_t below the value given by ⑤, $\dot{m}$ drops. +10

Specifically because the flow is then sonic at the throat,

$$\boxed{\dot{m} = \rho_* c_* A_t} \text{, as given by ④ above} \quad ⑥$$

+10

(+20)

END

if they write

$\dot{m} = \rho_* c_* A_*$ +2