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Statistics 21 **Mr. Purves**

Fall 2002

Midterm II

Print your name

Sign your name

TA's Name

Section time

To get full credit, you must give reasons and/or show work.

Score:

1: 5

2: 5

3: 5

4: 10

5: 10

6: 2

7: 10

Total 47

(5) 1. A longitudinal study of income involves interviewing a group of 200 individuals every six months over a five year period. At the beginning of the study, one person in the group happened to earn exactly four times the average income for the group. After six months, the income of this person had increased by 10%, but everyone else's income remained the same. This one change increased the average income in the group (over its value at the beginning of the study) by 0.2 %.

let x = average income of group at beginning of study

x_0 = average income of the individuals with the individual earning $4x$ the average left out

$$x = \frac{199x_0 + (4x)(1)}{200}$$

$$200x = 199x_0 + 4x$$

$$196x = 199x_0$$

$$x_0 = \frac{196}{199}x$$

y = avg. income after 6 months

$$y = \frac{199x_0 + 1.1(4x)}{200}$$

$$y = \frac{196x + 4.4x}{200} = \frac{200.4}{200}x$$

$$\text{change in avg income } (\%) = \frac{\frac{200.4}{200}x - x}{x} \cdot 100\% = \boxed{0.2\%}$$

5

5/15

(5) 2. Three dice, one red, one white, and one blue, are rolled out on a table.

(a) Find the chance the total number of spots rolled on the red and white dice is 8.

5 ways to get the sum of 8

Red	White
2	6
3	5
4	4
5	3
6	2

out of 6^2 total possible outcomes

$$\frac{5}{6^2} = \boxed{\frac{5}{36} \approx 13.9\%}$$

+2

(b) The three dice are picked up and rolled again. Find the chance the total number of spots rolled on the white and blue dice is 8.

The probability here is the same as in a) at $\boxed{\frac{5}{36}}$ since color itself has no bearing on the sum.

+1

(c) The dice are picked up and rolled a third time. Find the chance that both totals—the total for the red and white dice and the total for the white and blue dice—are 8.

* mutually exclusive

multiple ways this can happen

i)	if white is	2	blue & red must be 6
ii)	" "	3	" " 5
iii)	" "	4	" " 4
iv)	" "	5	" " 3
v)	" "	6	" " 2

prob. i) = prob. ii) = ... = prob v)

chance (sum for both pairs is 8) = prob i) + prob ii) + ... + prob v)

= 5 prob i)

$$= 5\left(\tfrac{1}{6}\right)\left(\tfrac{1}{6}\right)\left(\tfrac{1}{6}\right) = \boxed{\frac{5}{216} \approx 2.315\%}$$

+2

(5) 3. Fred and Henry both toss a coin three times a week. Fred does this for 50 weeks; Henry for 100 weeks.

It might turn out that:

(i) Fred gets three heads in more than 6 of the 50 weeks.

(ii) Henry gets three heads in more than 12 of the 100 weeks.

5

Check (√) one of the three options below, and then explain your choice.

____ (i) is more likely than (ii).

____ (i) and (ii) are equally likely.

__√__ (i) is less likely than (ii).

To explain my answer I will show the work that led to it.

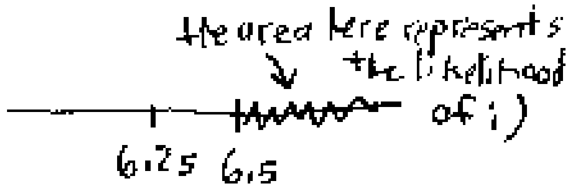
- The box used is $\lfloor 1 \cdot \boxed{1}$ and $7 \times \boxed{0} \rfloor$ where 1 is drawn as often as 3 heads are flipped in one week ✓

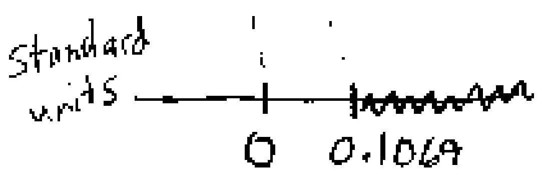
For Fred

$EV = \frac{1}{8} \cdot 50 = 6.25$ wks of 3 heads ✓

$SE = \sqrt{50} \cdot \sqrt{\frac{1}{8} \cdot \frac{7}{8}}$

≈ 2.3385 ✓ wks of 3 heads



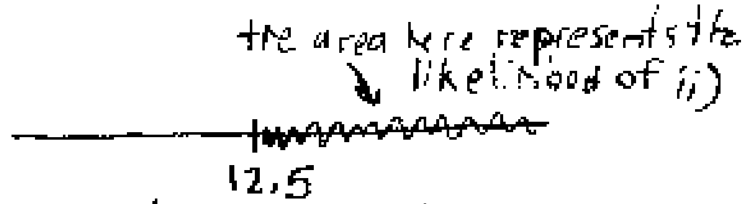


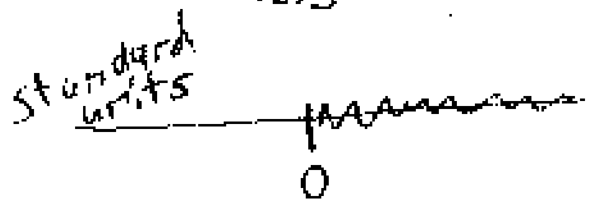
For Henry

$EV = \frac{1}{8} \cdot 100 = 12.5$ wks of 3 heads ✓

$SE = \sqrt{100} \cdot \sqrt{\frac{1}{8} \cdot \frac{7}{8}}$

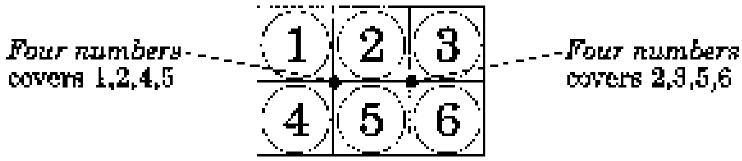
≈ 3.3072 wks of 3 heads ✓





From this work it can be seen that the chance of Fred getting 3 heads in more than 6 weeks is represented by the area to the right of 0.1069 under the normal curve and the chance of Henry getting 3 heads in more than 12 weeks is represented by the area to the right of 0 under the normal curve. Therefore (i) is less likely than (ii). ✓

(10) 4. A gambler playing roulette can make a *four numbers* bet–for example, bet \$1 on the numbers 1, 2, 4, and 5:



The bet pays 8 to 1: if one of the four numbers comes up, the gambler gets the \$1 back, along with with an additional \$8; otherwise the gambler loses the \$1 stake.

A roulette wheel at Nevada has 38 slots, in each of which the ball is equally likely to fall. The slots are numbered 0, 00, 1, 2, . . . 35, 36.

A couple plays at the same roulette wheel. On every spin, both husband and wife make a four numbers bet. The husband always bets \$1 on 1, 2, 4, 5; and the wife \$1 on 2, 3, 5, 6. They do this for twenty spins.

(a) The net gain to the husband from the 20 ~~bets~~ spins will be like   4

the sum of ___20___ draws from the box:

$\boxed{\$8}$ $\boxed{\$8}$ $\boxed{\$8}$ $\boxed{\$8}$ and 34 of $\boxed{-\$1}$

38 tickets total

Draw a box in the space above. Indicate how many tickets are in the box, and what is written on each ticket.

(b) The net gain to the couple from the 20 ~~bets~~ spins will be like   6

the sum of ___20___ draws from the box:

both lose

$\boxed{\$7}$ $\boxed{\$7}$ $\boxed{\$7}$ $\boxed{\$7}$ $\boxed{\$16}$ $\boxed{\$16}$ and 32 of $\boxed{-\$2}$

38 tickets total   both win

only 1 of them wins

Draw a box in the space above. Indicate how many tickets are in the box, and what is written on each ticket.

(10) 5. Fifty draws will be made at random, with replacement, from the box:

−2	−1	0	1	2

Find, approximately, the chance the sum of the draws will be equal to 3 or −3.

Use Normal curve appx

$EV = (\#\text{draws})(\text{avg of box})$

$= 50 \cdot 0 = 0$

avg. box = 0

$SE = \sqrt{\#\text{draws}} \cdot SD(\text{box})$

$= \sqrt{50} \cdot \sqrt{2} = \sqrt{100} = 10$

$SD(\text{box}) = \sqrt{\dfrac{(-2)^2+(-1)^2+0^2+1^2+2^2}{5}}$

$= \sqrt{\dfrac{10}{5}} = \sqrt{2}$

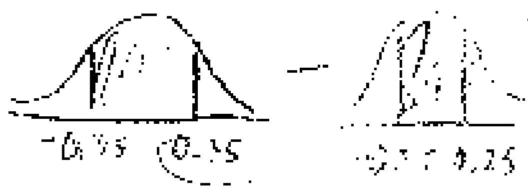
want to find probability sum falls in following ...

−3.5 −2.5 0 2.5 3.5

Standard units

−0.35 −0.25 0 0.25 0.35

10



$= (27.37 - 19.74)\%$

$\boxed{= 7.63\%}$

(5) 6. A large company hires an outside survey organization to interview a sample of its customers. The company wants the survey organization to use a probability method to obtain the sample. This requires the survey organization to satisfy two conditions when choosing the customers for the sample. State the two conditions.

1) The probability of selecting any two customers from the interview should be the same. No selection bias.

2) No customer may be chosen more than once, ie the customers surveyed must be "drawn without replacement".

(10) 7. A cable company takes a simple random sample of 350 households from a city with 37,000 households. Part of the data collected by the company is a list of the 350 households in the sample and the number of TV sets in each household. The total of this list is 637; that is, there is a total of 637 TV sets in the 350 sample households. The SD of the list is 0.75.

10

(a) Estimate the average number of TV sets per household in the city.

= avg # TV sets in sample household = $\frac{637}{350}$ $\boxed{= 1.82 \text{ TVs/household}}$

(b) Attach an SE to the estimate in (a).

$SE(\text{sum}) = \sqrt{\#\text{ sampled}} \cdot SD(\text{list})$

$= \sqrt{350} \cdot 0.75$

$SE(\text{avg.}) = \frac{SE(\text{sum})}{\#\text{ sampled}} = \frac{\sqrt{350} \cdot 0.75}{350}$ $\boxed{\approx 0.04 \text{ TVs/household}}$

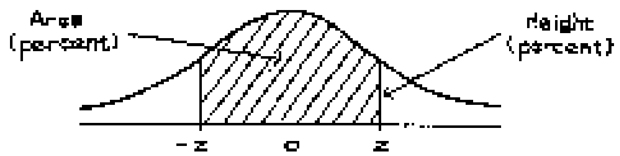
(c) In the statement below, fill in the two blanks with numbers.

The range from 1.74 to 1.9 is an approximate 95%-confidence interval for the average number of TV sets per household in the city.

95% confidence interval $\Leftrightarrow$ est. avg. ± 2 SEs

$= 1.82 \pm 2(0.04)$

Table



A NORMAL TABLE

z	*Height*	*Area*	*z*	*Height*	*Area*	*z*	*Height*	*Area*
0.00	39.89	0	1.50	12.95	86.64	3.00	0.443	99.730
0.05	39.84	3.99	1.55	12.00	87.89	3.05	0.381	99.771
0.10	39.69	7.97	1.60	11.09	89.04	3.10	0.327	99.806
0.15	39.45	11.92	1.65	10.23	90.11	3.15	0.279	99.837
0.20	39.10	15.85	1.70	9.40	91.09	3.20	0.238	99.863
0.25	38.67	19.74	1.75	8.63	91.99	3.25	0.203	99.885
0.30	38.14	23.58	1.80	7.90	92.81	3.30	0.172	99.903
0.35	37.52	27.37	1.85	7.21	93.57	3.35	0.146	99.919
0.40	36.83	31.08	1.90	6.56	94.26	3.40	0.123	99.933
0.45	36.05	34.73	1.95	5.96	94.88	3.45	0.104	99.944
0.50	35.21	38.29	2.00	5.40	95.45	3.50	0.087	99.953
0.55	34.29	41.77	2.05	4.88	95.96	3.55	0.073	99.961
0.60	33.32	45.15	2.10	4.40	96.43	3.60	0.061	99.968
0.65	32.30	48.43	2.15	3.96	96.84	3.65	0.051	99.974
0.70	31.23	51.61	2.20	3.55	97.22	3.70	0.042	99.978
0.75	30.11	54.67	2.25	3.17	97.56	3.75	0.035	99.982
0.80	28.97	57.63	2.30	2.83	97.86	3.80	0.029	99.986
0.85	27.80	60.47	2.35	2.52	98.12	3.85	0.024	99.988
0.90	26.61	63.19	2.40	2.24	98.36	3.90	0.020	99.990
0.95	25.41	65.79	2.45	1.98	98.57	3.95	0.016	99.992
1.00	24.20	68.27	2.50	1.75	98.76	4.00	0.013	99.9937
1.05	22.99	70.63	2.55	1.54	98.92	4.05	0.011	99.9949
1.10	21.79	72.87	2.60	1.36	99.07	4.10	0.009	99.9959
1.15	20.59	74.99	2.65	1.19	99.20	4.15	0.007	99.9967
1.20	19.42	76.99	2.70	1.04	99.31	4.20	0.006	99.9973
1.25	18.26	78.87	2.75	0.91	99.40	4.25	0.005	99.9979
1.30	17.14	80.64	2.80	0.79	99.49	4.30	0.004	99.9983
1.35	16.04	82.30	2.85	0.69	99.56	4.35	0.003	99.9986
1.40	14.97	83.85	2.90	0.60	99.63	4.40	0.002	99.9989
1.45	13.94	85.29	2.95	0.51	99.68	4.45	0.002	99.9991