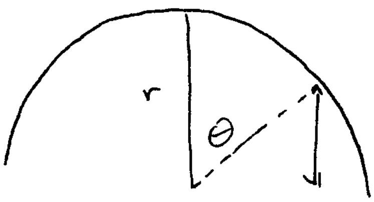
Yildiz MT 2

#1/ (a) We will use conservation of energy

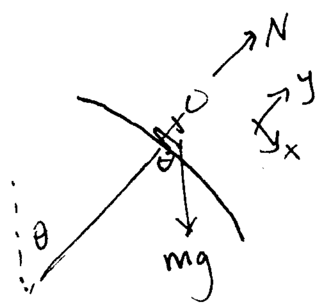


$h = r\cos\theta$

$E_i = mgr$

$E_\theta = mgr\cos\theta + \frac{1}{2}mv^2 \longrightarrow \underline{v^2 = 2gr(1-\cos\theta)}$ ①

At θ, draw FBD



$\sum F_y = N - mg\cos\theta = -\frac{mv^2}{r}$

The skier loses contact when $N = 0$

$\Rightarrow mg\cos\theta = \frac{mv^2}{r} \longrightarrow v^2 = rg\cos\theta$ ②

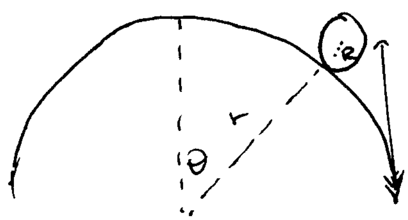
Combining ① & ②

$rg\cos\theta = 2gr(1-\cos\theta) \longrightarrow 3\cos\theta = 2$

$\cos\theta = 2/3$ or $\boxed{\theta = 48.2^\circ}$

Yildiz MT 2

#1 (b) Now we have a ring, $I_{ring} = mR^2$



$h = (r+R)\cos\theta$

Again we use energy conservation

$E_i = mg(r+R)$

$E_\theta = mg(r+R)\cos\theta + \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I\omega^2$

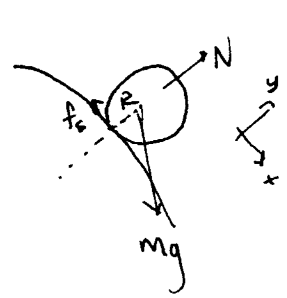
Since it rolls without slipping, $\omega = V_{cm}/R$.

Plug in I_{ring} & set $E_i = E_\theta$:

$$mg(r+R) = mg(r+R)\cos\theta + \frac{1}{2} m V_{cm}^2 + \frac{1}{2} mR^2 \left(\frac{V_{cm}}{R}\right)^2$$

$$\Rightarrow V_{cm}^2 = g(r+R)(1-\cos\theta). \quad ①$$

FBD:



$$\Sigma F_y = N - mg\cos\theta = -\frac{mV_{cm}^2}{r+R} \xrightarrow{N=0} V_{cm}^2 = (r+R)g\cos\theta \quad ②$$

Combine ① & ②

$$\cancel{g}\cancel{(r+R)}(1-\cos\theta) = \cancel{(r+R)}\cancel{g}\cos\theta$$

$$\cos\theta = 1/2 \rightarrow \boxed{\theta = 60^\circ}$$

Yildiz midterm 2
Problem 2.

Solution:

(1) For part (a) and (b), no friction

$$E_k + E_p = \text{constant}.$$

$$\Delta E_k + \Delta E_p = 0 \Rightarrow \Delta E_p = -\Delta E_k = 0$$

So we have $E_{pi} = E_{pf}$.

$$E_{pi} = \frac{1}{2}k(\Delta x_0)^2 = \frac{1}{2}\times 80\times 0.5^2 = 10\,(J).$$

(a) $E_{pf} = mgL\sin\theta$

so $$L = \frac{E_{pf}}{mg\sin\theta} = \frac{E_{pi}}{mg\sin\theta} = \frac{10}{1.8\times 9.8\times 0.6} = 0.9\,(m)$$

(b) $$E_{pf} = mgL\sin\theta + \frac{1}{2}k(L-\Delta x_0)^2$$

where Δx_0 is the initial compression

$$\Delta x_0 = 1.00\,m - 0.5\,m = 0.5\,m$$

So $$E_{pi} = E_{pf} = mgL\sin\theta + \frac{1}{2}k(L-\Delta x_0)^2$$

Plug in numbers, we get

$$10 = 1.8\times 9.8\times 0.6\,L + \frac{1}{2}\times 80\times (L-0.5)^2$$

$$\Downarrow$$

$$40L\left(L - 1 + \frac{1.8\times 9.8\times 0.6}{40}\right) = 0$$

$L \neq 0$ (we are interested in the solution $L > 0.5m$)

So $L = 1 - \frac{1.8\times 9.8\times 0.6}{40} = 0.7\,(m)$

(2) For part (c)

$$W_f = \Delta E_k + \Delta E_p$$

$$\Delta E_k = 0$$

$$\Delta E_p = \Delta E_{p,s} + \Delta E_{p,g}$$

$$= -\frac{1}{2}k(\Delta X_0)^2 + mg\,\Delta X_0 \sin\theta.$$

So $W_f = -\frac{1}{2}k(\Delta X_0)^2 + mg\,\Delta X_0 \sin\theta.$

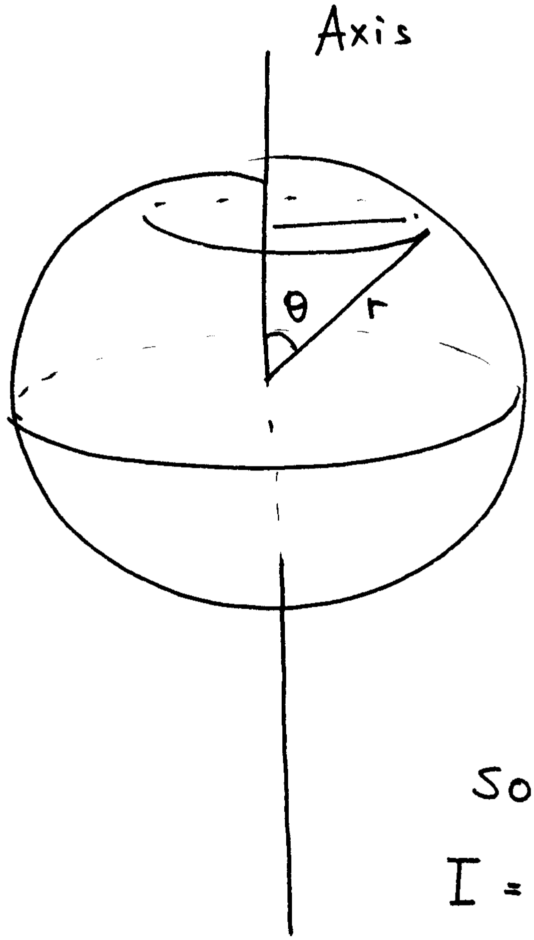
On the other hand. $W_f = -\mu_k mg\cos\theta\,\Delta X_0$

So $-\mu_k mg\cos\theta\,\Delta X_0 = -\frac{1}{2}k(\Delta X_0)^2 + mg\,\Delta X_0 \sin\theta.$

$$\mu_k = \frac{\frac{1}{2}k\Delta X_0 - mg\sin\theta}{mg\cos\theta} = \frac{\frac{1}{2}\times 80\times 0.5 - 1.8\times 9.8\times 0.6}{1.8\times 9.8\times 0.8} = 0.7$$

3

a)



$$\int (r\sin\theta)^2\, dm = I .$$

$$\begin{cases} dm = \rho\, dV , \quad \rho = M \big/ \frac{4}{3}\pi R^3 . \\ dV = r^2 \sin\theta\, dr\, d\theta\, d\phi . \end{cases}$$

so,

$$I = \rho \int_0^{R_0} dr \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \; r^4 \sin^3\theta$$

$$= 2\pi\rho \, \frac{R_0^5}{5} \int_0^{\pi} d\theta \, \sin^3\theta$$

$$\int_0^{\pi} d\theta \, \sin^3\theta = \int d\cos\theta \, (\cos^2\theta - 1)$$

$$= 4/3 .$$

thus, $$I = 2\pi \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{R_0^5}{5} \times 4/3 = 2/5 \, M R_0^2$$

3

b)

Without slipping : $R_0\omega = v$.

and

total kinetic energy

$$= (\text{translational}) + (\text{rotational})$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2} \times \frac{2}{5} MR_0^2\,\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2.$$

at the highest point in the circle,

$$V = h_{cm}Mg = (2R - R_0)Mg.$$

due to the energy conservation,

$$Mg(2R + Y - R_0) = (2R - R_0)Mg + \frac{7}{10}Mv^2$$

$$\therefore\ MgY = \frac{7}{10}Mv^2.$$

and thus, $v^2 = 10/7\, gY$.

not to fall at the higlest point,

$$\cancel{M}\frac{v^2}{R - R_0} \geq \cancel{M}g \quad \therefore\ Y \geq \frac{7}{10}(R - R_0).$$

Yildiz MT 2 #4

The equation we need is

$$M \frac{d\vec{v}}{dt} = \sum \vec{F}_{ext} + \vec{v}_{rel} \frac{dM}{dt}$$

The external force is due to gravity:

$$F_g = -\frac{M_E m G}{(R_E + h)^2} = -\frac{M_E m G}{4 R_E^2} = -m\frac{g}{4}$$

(the altitude h is numerically equal to R_E for this problem)

where $g = \frac{G M_E}{R_E^2} \sim 10\ m/s^2$. Let $m = 2.4 \cdot 10^4\ kg$, $a = 1.5\ m/s^2$, $v_{rel} = 1.3\ \frac{km}{s}$

and $R = \left| \frac{dM}{dt} \right|$. Our equation is now just

$$ma = \frac{-mg}{4} + v_{rel} R$$

$$\rightarrow R = \frac{m(a + g/4)}{v_{rel}} = \boxed{76.9\ kg/s}$$

(5) (a) Linear momentum is conserved,

$$\left.\begin{aligned}&\Rightarrow mv_0 = mv_f + mv_{cm}\\&\Rightarrow v_{cm} = v_0 - v_f \quad -(1)\end{aligned}\right\} \text{2 Marks}$$

Angular momentum is conserved, → 1 mark

$\Rightarrow mv_0 l = I_{cm}\omega_f + mv_f l$ → 1 mark

$I_{cm} = \frac{M(2l)^2}{12} = \frac{Ml^2}{3}$ → 1 Mark

$\Rightarrow v_0 = \frac{\omega_f l}{3} + v_f \Rightarrow \omega_f l = 3(v_0 - v_f) \quad -(2)$

Energy is conserved,

$$\left.\begin{aligned}&\Rightarrow \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}I_{cm}\omega_f^2 + \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}mv_{cm}^2\\&\Rightarrow v_0^2 = \frac{(\omega_f l)^2}{3} + v_f^2 + v_{cm}^2 \quad -(3)\end{aligned}\right\} \text{3 Marks}$$

Substituting (1), (2) in (3), we get $v_f = v_0, \frac{3v_0}{5}$ } 4 Marks for math and final answer

neglected

Hence $\boxed{v_f = \frac{3v_0}{5}}$

(b) As the rod is fixed, Linear momentum is not conserved and rotation takes abt the pivoted point.

Hence I to be considered : $I_{end} = \frac{M(2L)^2}{12} + M(L)^2 = \frac{4ML^2}{3}$ — 1 Mark

→ Angular momentum is conserved,

$$\left.\begin{aligned}&mv_0(2l) = I_e\omega_f + mv_f(2l)\\&\Rightarrow \omega_f l = \frac{3}{2}(v_0 - v_f) \quad -(4)\end{aligned}\right\} \text{2 Marks}$$

→ Energy is conserved,

$$\left.\begin{aligned}&\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I_e\omega_f^2\\&\Rightarrow v_0^2 = v_f^2 + \frac{4}{3}(\omega_f l)^2 \quad -(5)\end{aligned}\right\} \text{2 Marks}$$

Solving for v_f from (4) and (5), we get } 3 Marks for Math and final answer.

$\boxed{v_f = \frac{v_0}{2}}$