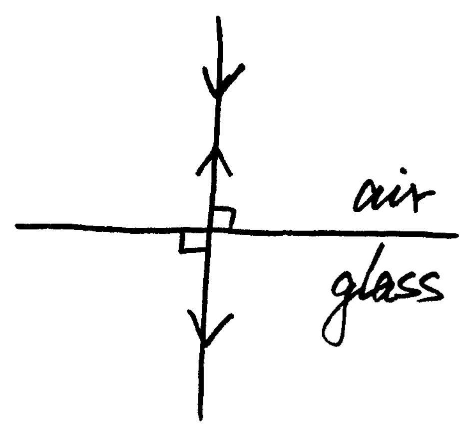
Problem 1

Solution

(a)

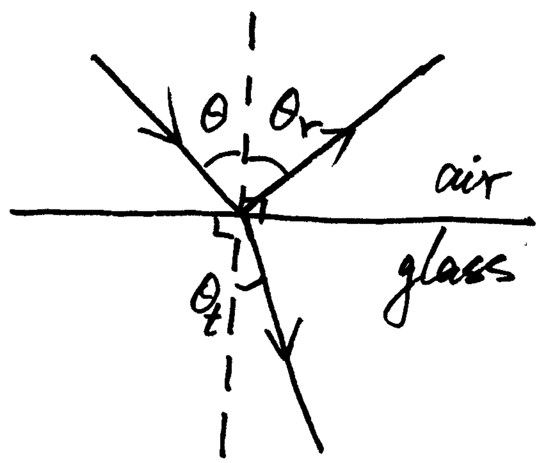
$\theta = 0^\circ$



reflected direction: $\theta_r = 0^\circ$

transmitted direction: $\theta_t = 0^\circ$

$\theta = 45^\circ$



reflected direction: $\theta_r = 45^\circ$

transmitted direction: $\cancel{\theta_t}$

$\theta_t = \arcsin \frac{\sqrt{2}}{3}$.

$n_{air} \sin\theta = n_{glass} \sin\theta_t$.

$1 \times \sin 45^\circ = 1.5 \times \sin\theta_t$

$\sin\theta_t = \frac{2}{3} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{3}$

(b) TIR is Not possible for light incident from the air side.

(c) Yes. TIR is possible for light incident from the glass side.

$n_{glass} \sin\theta \geq n_{air} = 1$.

$\therefore \sin\theta \geq \frac{1}{n_{glass}} = \frac{1}{1.5} = \frac{2}{3}$.

$\theta \geq \arcsin\frac{2}{3}$, of course $\theta \leq 90°$

(d) Light travels faster on the air side.

$V_{air} = \frac{c}{n_{air}} = \frac{c}{1} = c$

$V_{glass} = \frac{c}{n_{glass}} = \frac{c}{1.5} = \frac{2}{3}c$.

Rubric

(a) no reflected ray in one picture -1.

$\theta = 0^\circ \begin{cases} \theta_r = 0^\circ \\ \theta_t = 0^\circ \end{cases}$ θ_r、θ_t 1 point respectively

$\theta = 45^\circ \begin{cases} \theta_r = 45^\circ & 1 \\ \boxed{\theta_t = \arcsin\frac{\sqrt{2}}{3} \quad 2 = 1 + 1} \end{cases}$

snell law value.

qualitative describe will also get the credits.

(b) 5

(c) $\theta_c = \arcsin\frac{2}{3}$ 4.

$\theta \geq \theta_c$ 1 ($\theta > \theta_c$ cannot get this point)

(d) air side faster 1

$V = \frac{c}{n}$ 2.

values 1+1

Problem 2

Solution

(a)

$$\Delta X = \lambda \frac{L}{d}.$$

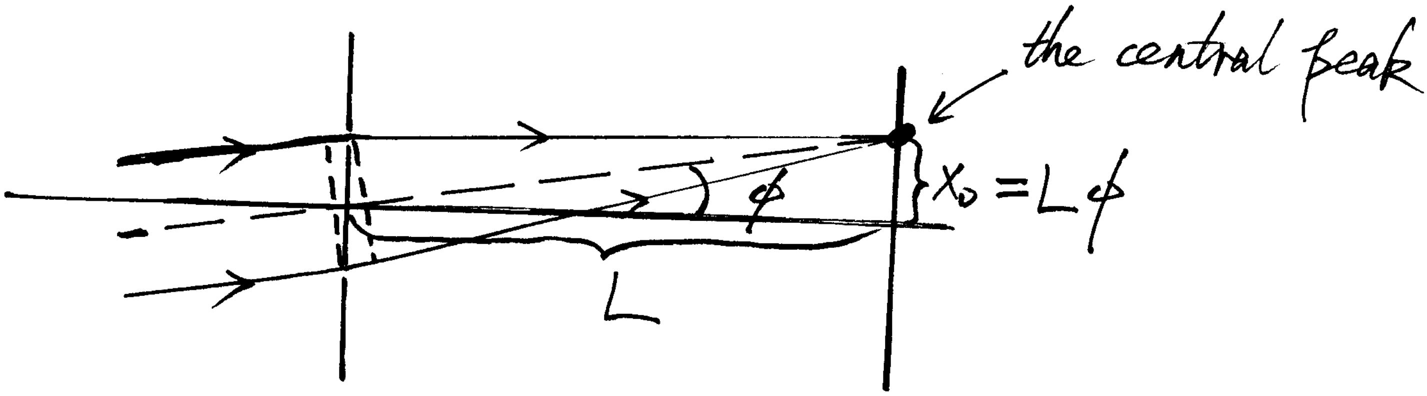
$\lambda = 1cm = 0.01m$

$L = 100m$

$\Delta X = 1m$

$$d = \frac{\lambda L}{\Delta X} = \frac{0.01 \times 100}{1} = 1 (m)$$

(b)



the pattern is shifted by $X_0 = L\phi$.

(c) $\frac{d}{D} = 3$ (actually $\frac{d}{D} = \frac{3}{n}$

$n = 1$ or 2

but most students assumed $n=1$, so...)

Rubric

(a)

$$\left.\begin{array}{l}\Delta\varphi = \frac{2\pi}{\lambda} d\sin\theta \approx \frac{2\pi}{\lambda} d\theta \\ \Delta\varphi = m\cdot 2\pi\end{array}\right\} \Rightarrow d\theta = m\lambda$$

$$\theta \approx \frac{x}{L} \Rightarrow d\frac{x}{L} = m\lambda$$

$$\Delta x \frac{d}{L} = \Delta m\cdot\lambda = \lambda$$

$$\Downarrow$$

$$\Delta x = \lambda\frac{L}{d}$$

Or equivalent deduction process to get this relation : 6 points

$\Delta x = 2\lambda\frac{L}{d}$ -3

No deduction but wrote down $d = \frac{\lambda L}{\Delta x}$ and with correct final value of d : 10 points.

Making mistakes when plugging numbers into correct equation : -2.

(b) picture 2.

shifted only 1

the amount of shift $X_0 = L\phi$ 2.

(c) $\boxed{\text{missing order } m=3}$ if wrong −4 points

$\boxed{\frac{d}{D} = m}$ if wrong −4 points

those are just in case that the final answer is incorrect

But if correct final answer $\frac{d}{D} = 3$ is got, then full points.

Problem 3

Solution

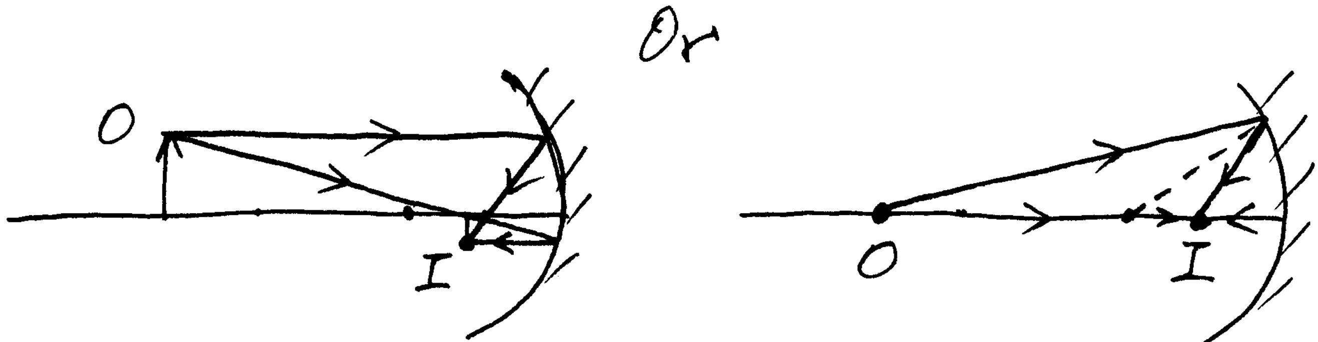
(a) $d_o \to \infty$, $d_i = 1m$.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{1}$$

$$f = 1m$$

$$f = \frac{R}{2}, \quad R = 2f = 2m.$$

(b)



Real Image.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow \frac{1}{5} + \frac{1}{d_i} = \frac{1}{1} \Rightarrow d_i = \frac{5}{4}m$$

(c) $\frac{1}{5} + \frac{1}{d_i} = \frac{1}{-1} \Rightarrow d_i = -\frac{5}{6}m$

Virtual Image.

Rubric

(a) $f = 1\,m$ 3 points

$R = 2\,m$ 2 points

if final answer incorrect, then

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ 2 points

$f = \frac{R}{2}$ 1 point.

(b) picture 5 points

real image 5 points

$d_i = \frac{5}{4}\,m$ 5 points

if value of d_i incorrect, then

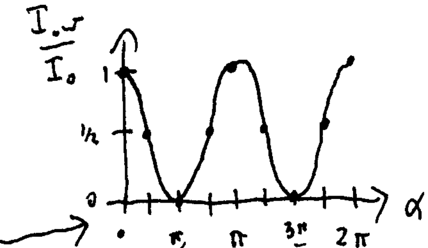
$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ 3 points.

(c) $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ 2 points

$d_i = -\frac{5}{6}\,m$ 1 point virtual image 2 points

Solutions and Rubric for #4:

4.(a) Equation for solution is $I_{out} = I_0 \cos^2 \alpha$, where α is the relative angle between polarization axis and $\hat{x}$. (+5 pts for equation)



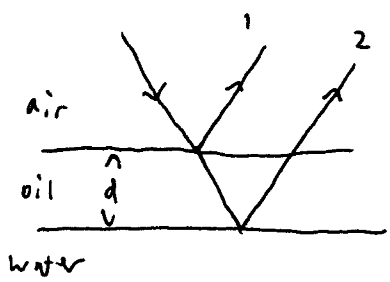
+4 pts for graph (with correct axes, units).

I also accept plots against absolute angle, so long as you specify initial orientation of pol. axis, or indication of knowledge thereof, depicted in graph.

Only graph $[0, 2\pi)$, per directions!

$I_{max} = I_0$, $I_{min} = 0$ (+1 pt.)

4(b)



(angles in figure are simply schematic)

$\phi_2 - \phi_1 = 2\pi m$ (+1 for indication of condition for constructive interference, in any valid fashion)

$\phi_1 = \pi$ since $n_0 > 1$ (+2 for identification that ray 1 undergoes phase change due to reflection)

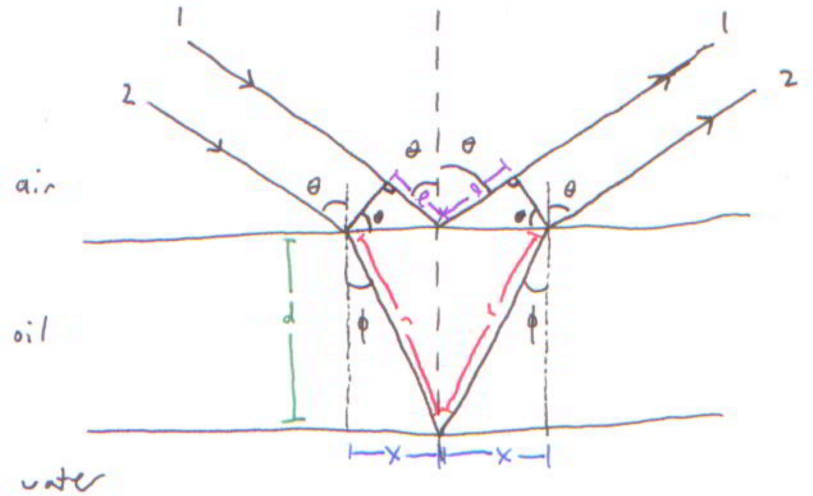
$\phi_2 = \pi + 2d \frac{2\pi n_0}{\lambda}$ (+2 for ray 2 phase change upon reflection, +2 for correct phase due to path traveled. One point taken off for forgetting "n_0") another point off for forgetting factor of 2.

$\rightarrow d = \frac{1}{2} \lambda n_0^{-1} m$

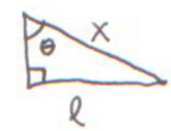
for $m = 1, 2, \ldots$

(+1 for correct final result, including correct values for m)

4.c Let's try this again ...



Zoom-in on triangle with "ℓ": (after rotating)



$$\text{Snell: } \sin\theta = n_0 \sin\phi \rightarrow \sin\phi = \frac{1}{n_0}\sin\theta$$

$$\phi_1 = 2\ell\,\frac{2\pi}{\lambda} + \pi \; ; \quad \phi_2 = 2r\,\frac{2\pi n_0}{\lambda} + \pi .$$

Useful trig: $\sin\theta = \frac{\ell}{x}$, $\cos\phi = \frac{d}{r}$, $\tan\phi = \frac{x}{d}$

$$\rightarrow \ell = x\sin\theta = d\sin\theta\tan\phi = \frac{d}{n_0}\frac{\sin^2\theta}{\sqrt{1-\frac{1}{n_0^2}\sin^2\theta}} ,$$

where $\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\frac{1}{n_0^2}\sin^2\theta}$.

$$r = \frac{d}{\sqrt{1-\frac{1}{n_0^2}\sin^2\theta}}$$

$$\phi_2 - \phi_1 = \frac{2\pi}{\lambda}\cdot 2\cdot\left(n_0\frac{d}{\sqrt{1-\frac{1}{n_0^2}\sin^2\theta}} - \frac{1}{n_0}\, d\,\frac{\sin^2\theta}{\sqrt{1-\frac{1}{n_0^2}\sin^2\theta}}\right)$$

$$= \frac{2\pi}{\lambda}\, 2n_0 d\,\frac{1}{\sqrt{1-\frac{1}{n_0^2}\sin^2\theta}}\left(1-\frac{1}{n_0^2}\sin^2\theta\right)$$

$$\Rightarrow \frac{2\pi}{\lambda}\, 2n_0\, d\sqrt{1-\frac{1}{n_0^2}\sin^2\theta} = 2\pi m$$

$$\hookrightarrow \boxed{d = \frac{\lambda}{2n_0}\frac{1}{\sqrt{1-\frac{1}{n_0^2}\sin^2\theta}}\, m ; \quad m = 1,2,3,\ldots}$$