First Midterm, Math 54, 2010022416-17.

Your name: ______________________

Student ID #: ______________________

GSI & meeting time: ______________________

1	
2	
3	
4	

Correct answer without explanation = no credit

① Find all values of a such that $\begin{bmatrix} a & 1 & & \\ 1 & 2 & 1 & \\ & 1 & 2 & 1 \\ & & 1 & a \end{bmatrix}$ is singular.

① $\det \begin{bmatrix} a & 1 & & \\ 1 & 2 & 1 & \\ & 1 & 2 & 1 \\ & & 1 & a \end{bmatrix} = a \begin{vmatrix} 2 & 1 & \\ 1 & 2 & 1 \\ & 1 & a \end{vmatrix} - 1 \begin{vmatrix} 1 & & \\ & 2 & 1 \\ & 1 & a \end{vmatrix} = a(4a-2-a) - (2a-1)$

$= 3a^2 - 4a + 1 = 0 \Rightarrow a = \dfrac{4 \pm \sqrt{4^2 - 3\cdot 4}}{2\cdot 3} = \dfrac{4 \pm 2}{6} = \begin{cases} 6/6 = 1 \\ 2/6 = 1/2. \end{cases}$

(2) Let $A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 0 & 2 & 3 \\ 2 & 0 & 3 & 5 \end{bmatrix}$.

(a) Find a basis for Nul (A)

(b) Find a basis for Col (A).

(c) Let $T\underline{x} = A\underline{x}$ map $\mathbb{R}^4$ into $\mathbb{R}^3$. Is T onto?

(2) $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 0 & 2 & 3 \\ 2 & 0 & 3 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a) Free parameters: $x_2 = s_1$, $x_4 = s_2 \Rightarrow x_1 + x_4 = 0$ and $x_3 + x_4 = 0 \Rightarrow$

$\underline{x} = \begin{bmatrix} -s_2 \\ +s_1 \\ -s_2 \\ s_2 \end{bmatrix} = s_1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s_2 \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ so null (A) has basis $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

(b) The pivot columns are (1), (3) Hence basis for $\mathrm{Col}(A) = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$.

(c) Since $\dim \mathrm{Col}(A) = 2$, $A\underline{x}$ cannot fill $\mathbb{R}^3$. Hence T is not onto.

indeed there is no x such that $A\underline{x} = \underline{e}_3$.

③ Let $H = \{ p(t) \in \mathbb{P}_2 : (\frac{d}{dt} p)(1) = 0 \}$

(a) Show that H is a subspace of $\mathbb{P}_2$.

(b) Find a basis for H.

③ If $p'(1) = q'(1) = 0 \Rightarrow (p+q)'(1) = 0$ and $(c \cdot p')(1) = c\,p'(1) = 0$. and $p \equiv 0$ implies that $p'(1) = 0$. Hence H is a subspace.

(b) Assume $p = a + bt + ct^2 \Rightarrow p'(1) = 0 + b + 2c \cdot 1 = b + 2c = 0 \Rightarrow b = -2c$

$\Rightarrow p = a - 2ct + ct^2 = a + c(t^2 - 2t)$. So $\mathcal{B} = \{1, t^2 - 2t\}$.

Are linearly indep.: because if $a + c(t^2 - 2t) = 0 \quad \forall t \Rightarrow t = 0 : a = 0 \Rightarrow c(t^2 - 2t) = 0 \Rightarrow c =$

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