$\qquad$ SOLUTION $\qquad$

# UNIVERSITY OF CALIFORNIA, COLLEGE OF ENGINEERING E77: INTRODUCTION TO COMPUTER PROGRAMMINGFOR SCIENTISTS AND ENGINEERS 

Professor Raja Sengupta
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2nd Midterm Exam—April 11, 2007


| Question | Points | Grade |
| :---: | :---: | :---: |
| Part I | 5 |  |
| Part II | 2 |  |
| Part III | 8 |  |
| Part IV | 10 |  |
| Part V | 5 |  |
| TOTAL | 30 |  |

Notes:

1. Write your name/SID on the top of every page and your signature below.
2. Please give all your answers only in the spaces provided.
3. You may NOT ask any questions during the exam.
4. You may NOT leave the exam room before the exam ends.

Your signature: $\qquad$
Your E77 LECTURE SECTION 1(12-1pm) or 2(1-2pm) (Circle your section \#)
Circle your Lab Section (where the graded midterms will be returned).

| \#11: MW 8-10 | \#12: MW 10-12 | \#13: MW 2-4 |
| :---: | :---: | :---: |
| \#14: MW 4-6 | \#15: TuTh 8-10 | \#16: TuTh 10-12 |
| \#17: TuTh 12-2 | \#18: TuTh 2-4 | \#19: TuTh 4-6 |

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## PART I Questions 1-5: Systems of Linear Equations

Questions 1-5 refer to the following system of equations:
$-x+y+2 z=2$
$3 x-y+z=6$
$-x+3 y+4 z=4$

1. ( 1 point $)$ Write the matrices so that this system of equations in the form $A x=b$.

$$
A=\left[\begin{array}{ccc}
-1 & 1 & 2 \\
3 & -1 & 1 \\
-1 & 3 & 4
\end{array}\right] \square\left[\begin{array}{l}
2 \\
6 \\
4
\end{array}\right]
$$

2.(1 point) After correctly assigning the matrices $A$ and $b$ in Matlab, to solve this system of equations via Gaussian elimination we should type at the command prompt:
a. $\gg A \backslash b$
b. $\gg \operatorname{pinv(A)*b}$
c. $\gg \operatorname{inv}(A) * b$
d. $\gg A^{\prime} \mid b$
3.(1 point) If $\operatorname{det}(\mathrm{A})=0$
a. The system of equations have no solution.
b. The system of equations have a unique solution.
c. The system of equations have more than one solution.
d. The system of equations has either no solution or more than one solution.
4.(1 point) Consider the equations $2 x+4 y=5$ and $2 x+4 y=7$. The $\operatorname{rank}\left(\left[\begin{array}{ll}2 & 4 \\ 5 & 2\end{array} 47\right]\right)$ will be
a. 0
b. 1
c. 2
d. 3

Hint: $\operatorname{rank}([24 ; 24])=1$.
5.(1 point) Consider the equations $2 x+4 y+3 z=5,2 x+4 y+z=5$. To find a solution to these equations we can type:
a. $\operatorname{pinv}([243 ; 241]) *[5 ; 5]$
b. inv([2 4 3;2 4 1])*[5;5]
c. $\operatorname{pinv}([243 ; 241]) *[55]$

## PART II Questions 6-7: Computer Representation of Numbers

6.(1 point) The general form of an encoded number, according to IEEE 754, is:
a. $(-1)^{\text {sign }} * 2^{\text {Exponent }} *$ Fraction
b. $(-1)^{\text {sign }} * 2^{\text {biasedExponent }} *$ Fraction
c. $(-1)^{\text {sign }} *(-2)^{\text {Exponent }} *$ Fraction
d. $(1)^{\text {sign }} * 2^{\text {biasedExponent }} *$ Fraction
7.(1 point) How many bits are allocated to the biasedExponent for IEEE 754 single precision?
a. 127
b. 8
c. 7
d. 255
$\qquad$ SOLUTION $\qquad$

## PART III Questions 8-15: Regression and Interpolation


8.(1 point) Consider the graph shown in the above figure. What would be the Y value, using linear interpolation, corresponding to $\mathrm{X}=7.5$.
a. $Y=4.25$
b. $Y=4.5$
c. $Y=4.75$
d. $\mathrm{Y}=5$
9.(1 point) Consider the same graph. What would be the Y value, using cubic spline Interpolation, corresponding to $\mathrm{X}=5$.
a. $\mathrm{Y}=3$
b. $Y=3.25$
c. $Y=3.5$
d. $Y=4$
10.(1 point) If you were to estimate a regression model (named model1) on these data using the polyfit, what would be the slope coefficient and $y$ intercept, respectively?
a. $7 / 12,0$
b. 0,1
c. $6 / 12,0$
d. $6 / 12,1$
11.(1 point) If you specified a new model (named model2) on the same data set, but did not include an intercept (regression line passes through $(0,0)$ ), which of the following statements is true?
a. Sum of square errors (model1)<sum of square errors (model2)
b. Sum of square errors (model1)=sum of square errors (model2)
c.Sum of square errors (model 1)>sum of square errors(model 2)
$\qquad$ SOLUTION $\qquad$
12.(1 point) Suppose you have a vector x and a vector y and you typed $\mathrm{b}=$ polyfit( $x, y, 4$ ) at the Matlab command prompt and it returned:
b =
0.0014
0.2345
-0.1661
-2.1284
1.7561

Write out the polynomial function that these numbers represent $y=.0014 x^{4}+.2345 x^{3}-.1661 x^{2}-2.1284 x+1.7561$
13.(1 point) Which of the following statements are true for least squares:
a. The mean of the residuals is zero
b. The mean of the residuals squared is zero
c. The mean of the residuals should be the slope of the regression line
d. The mean of the residuals should be the intercept of the regression line
<Note that b. has an ambiguous statement: It could be interpreted as $\left(\frac{1}{n} \sum_{i} e_{i}\right)^{2}$ or as $\left(\frac{1}{n}\left(\sum_{i} e_{i_{i}}^{2}\right)\right)$ where $e_{i}$ is the residual of observation $i$. It was intended to mean the second of these formulae, however the following three answers are acceptable: (a) or (b) or (a and b) >
$\qquad$ SOLUTION $\qquad$

14. (1 point) Which of the lines in the above figure is the least squares regression line?
a. Line 1
b. Line 2
c. Line 3
d. Line 4
15. (1 point) Which of the lines in the above figure is the worst fit to the data in terms of the coefficient of determination?
a. Line 1
b. Line 2
c. Line 3
d. Line 4
$\qquad$ SOLUTION $\qquad$

## PART IV Questions 16-22: Root Finding

16.(1 point ) The roots of a function $f(x)$ are the values of $x$ upon solving the equation
a. $f(x)-1=0$
b. $\mathrm{f}(\mathrm{x})^{2}=0$
c. $f(x)=0$
d. None of the above.
17.(1 point) Consider the spring mass damper system $\ddot{x}+2 \dot{x}+1=0$ with initial conditions $x(0)=2, \dot{x}(0)=0$. Which of the following statements is true?
a. $x(t)$ has oscillations that die down over time.
b. $x(t)$ has no oscillations and the values die down over time.
c. $x(t)$ has oscillations that grow over time.
d. $x(t)$ has no oscillations but its value grows over time.
18.(1 point) Fill in the blank with a single Matlab command to find the roots of $2 x^{3}+3 x^{2}-5 x+1$.
>> $\qquad$ $\mathrm{R}=\operatorname{roots}([2,3,-5,1])$
19.(1 point) The polynomial $-x^{3}+2 x^{2}+30 x+1$ has a local maximum and minimum. Fill in the blank with a Matlab command to find the values of $x$ corresponding to the maximum and minimum.
$\gg \quad \mathrm{R}=\operatorname{roots}($ polyder $([-1,2,30,1]))$ or $\quad \mathrm{R}=\operatorname{roots}(-3,4,30)$
20.(1 point) fzero(' $1 / x^{\prime},-1,1$ ) will return a value near
a. 0
b. 1
c. -1
d. 0.5
21.(1 point) fzero(@tan,-0.5,0.2) will return a value near
a. -0.5
b. 0
c. 0.2
d. 1.57
22.(2 points) You are given the file myFunc. $m$ as shown below:
function $y=m y F u n c(x)$
$y=\exp (x)-2$;
end
Fill in the blank below with a single matlab command which will find its root:
>> $\qquad$ fzero(@(x) myfunc (x),0,10)
23.(3 points) Complete the following program so that it will find the root of func lying in the interval [lower, upper] to the precision specified by tol.
function root $=$ bisection(func, lower, upper, tol)
midpoint =(upper + lower)/2;
midFunc = feval(func, midpoint);
upperFunc =feval(func, upper);
if le(upper - lower, tol)
root $=$ midpoint ; elseif eq(sign(mid Func),sign(upperFunc))
root $=\quad$ bisection(func,lower,mid point,tol) $\quad$
else
root $=$
bisection(func, midpoint,upper,tol)
)
end
end
$\qquad$ SOLUTION $\qquad$

## PART V Questions 24-28: Numerical Differentiation and Integration

Answer the following questions using the below figure.
Assume you wish to numerically differentiate $\mathrm{F}(\mathrm{x})$ at $\mathrm{x}=-2$ to obtain approximations of the first derivative. You have data values at $x=-10,-6,-2,2$, and 6 . The true derivative at $x=-2$ is plotted in the graph.

24.(1 point) The $1^{\text {st }}$ order, forward-difference method derivative estimate will be $\qquad$ the true derivative at $\mathrm{x}=-2$.
a. greater than
b. less than
c. exactly the same as
25.(1 point) The $1^{\text {st }}$ order, backward-difference method derivative estimate will be the true derivative at $\mathrm{x}=-2$.
a. greater than
b. less than
c. exactly the same as
26.(1 point) The $2^{\text {nd }}$ order, central-difference method derivative estimate will be $\qquad$ the true derivative at $\mathrm{x}=-2$.
a. greater than
b. less than
c. exactly the same as
27.(1 point) The $4^{\text {th }}$ order central-difference method derivative estimate will be the true derivative at $\mathrm{x}=-2$.
a. greater than
b. less than
c. exactly the same as
28.(1 point) Assume you numerically integrate the function $f(x)=3 x^{2}+2 x+1$ using Simpson's rule integration on the interval [ 0,1 ], for points at $0, .25, .5, .75$, and 1 (i.e. you are trying to find the integral of $\mathrm{f}(\mathrm{x})$ from 0 to 1 using 4 subintervals).

The value you compute will be closest to:
a. 2
b. 2.9
c. $3 \leftarrow$ We know this immediately since Simpson's rule is exact for polynomials of degree $<=3$
d. 3.1
e. 4

