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BioE 153 MIDTERM October 7, 2002

Name ANS

Note: You may use calculators and back of these sheets but no notes or other materials. Answers are to be placed in the space provided. Some helpful and not helpful equations are attached. Place your name on each sheet now please. Good luck.

1. (15 pts)You wish to determine blood flow in the carotid artery using a Doppler ultrasound system operating at 5 MHz. The Doppler probe (transducer) is at an angle of 60 degrees relative to the orientation of the carotid artery along the trajectory of blood flow away from the probe. Calculate the velocity if the Doppler shift was 3.33 kHz.

$$v = \frac{-c \cdot \Delta f}{2 f_0 \cos\theta} = \frac{1500 \cdot 3.33\times10^{-3}\,MHz}{5\,MHz} = 1\,m/s$$

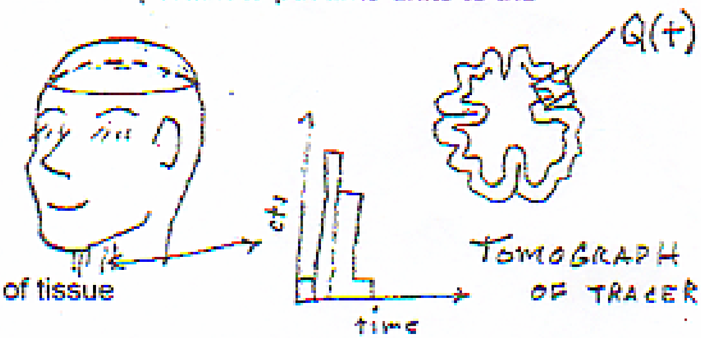
2. (20pts)Below is shown a transverse section from the human head wherein a quantitative measurement of blood flow is to be made. The cube shown is 1 cc of tissue whose concentration of a tracer can be determined by non invasive imaging. An injection is made into the vascular system of a tracer which flows into all tissues and during the circulation of the tracer through the body that which reaches the brain is trapped by enzymes in the brain parenchyma (in this case the gray matter).

   Two measurement sites are used. One is the concentration in the brain (Q(t)) and the other is the time varying concentration in the carotid artery. The carotid artery data are in cts per cc of blood. From the data below, calculate the flow in the brain assuming no tracer washes out of the volume, Q during the experiment. CALCULATE THE BLOOD FLOW IN THE BRAIN VOLUME IN TERMS OF cc/minute/cc of tissue.

   DATA

   Note: cts is a normalized unit which could by light photons per some time unit or radioactivity in counts per some time unit. It is not important to put time units to the measurement as it will confuse you.

Time(seconds)	Activity(cts)
0-10	10
10-20	100
20-30	80
30-40	10
40-50	0



At time 50 seconds Q(t=50) = 20 cts/cc of tissue

$$F = \frac{Q(T)}{\int A(t)\,dt} = \frac{Q(T=50)}{\sum A(t)\Delta t} = \frac{20}{100+1000+800+100}$$

$$= \frac{20}{2000} = 0.01\ cc/sec/cc$$

$$\boxed{\text{Per min} = 0.6\ cc/min/cc}$$

Name ANS

3.(15 pts) Why in the formulation of the conservation laws governing the flow of blood, or any other fluid, in the body did we first develop (i) the concept of the derivative following a particle (the so-called material derivative), and then introduce (ii) the idea of the rate of change of a volume integral "following the flow." (**Hint:** make your answer by less than 40 words recalling the conservation of mass and momentum. Do these laws apply to regions of space or to material (fluid) particles and what does you answer mean for the formulation of the conservation laws?)

i) Unlike physics where we observe the motion of a moving particle in a frame attached to the particle we make our observations in a fixed (Eulerian) Frame. In such a frame $a \neq \frac{\partial u}{\partial t}$ but it accounts acceleration due to motion in space.

ii) The conservation laws apply to a material volume of fluid i.e, a volume consisting of fluid particles, but such a material moves with fluid particles. So the statement of conservation laws require that for this material ~~[illegible]~~ volume, the overall (integral) of each quantity must be considered

4.(15 pts) If the flow of blood is to remain constant through the aorta the velocity of blood must increase from 26.5 cm/s to 40 cm/s as the radius decreases from 1 cm to 0.8 cm. Calculate the pressure difference between these two areas of the aorta in N/meter squared (Pascal).
(blood density is 1060 kg/meter cubed. 90 percent of the answer is in laying out the solution and not is getting an exact answer.

From continuity: $V_1 A_1 = V_2 A_2 \Rightarrow V_1(\pi r_1^2) = V_2(\pi r_2^2)$, ~~but this~~

$\Rightarrow V_1(\pi \times (0.1)^2) = 40(\pi \times (0.8)^2) \Rightarrow \boxed{V_1 = 25.6 \text{ cm/s}}$

typo in problem statement

In order to find pressure drop you should use bernoulli Equation

$$P_1 + \rho \frac{V_1^2}{2} + \cancel{\rho g z_1} = P_2 + \rho \frac{V_2^2}{2} + \cancel{\rho g z_2}$$

$\boxed{\text{For } V_1 = 26.5} \Rightarrow \boxed{P_1 - P_2 = 47.6 \text{ Pa}}$

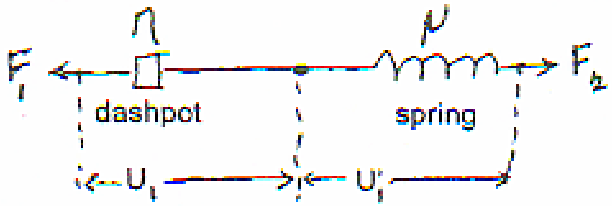
(4 pts)Two assumptions you make to arrive at a solution are that the flow is steady and the flow is along a streamline. Can you give 2 other assumptions?

① Inviscid Flow (No shear forces)

② Incompressible flow (ρ = const)
Rigid wall which result an incompressible flow

Name__________

5.(15 pts) Derive the constitutive equation for Maxwell model of viscoelastic material given the following: (Hint: recall the relationships between F and extensions and parameters of dashpot and springs).



U = total extension

η is the velocity of deflection for dashpot

μ is young's modulus for the spring (not ion mobility)

U_1 extension of dashpot

U_1': extension of spring

$$\left.\begin{aligned} U &= U_1 + u_1' \\ F_1 &= \eta \dot{u}_1 \\ F_2 &= \cancel{\mu \dot{u}_1'} \; \mu u' \end{aligned}\right\} \text{To be completed}$$

$$\dot{u}_1 = \frac{du}{dt} = \frac{F_1}{\eta} + \frac{\dot{F}}{\mu}$$

6.(15 pts) Consider a biologic cell which is permeable only to potassium ions (K+) and that the concentration of potassium is higher in the intracellular fluid than the extracellular fluid. Write down the equations which relate to diffusion due to the concentration gradient and the drift due to an electric field. Show the total flow. (Hint: This is almost trivial if you know what to look for so do not make it more than necessary).

$$J_{TOTAL} = J_{diffusion} + J_{elect} = -D\frac{d[K^+]}{dx} - \mu Z[K^+]\frac{dV}{dx}$$

7.(optional depending on which you replace) You may replace questions 1 through 5 with the answer to this which relies on the answer to 6. Using the Einstein relationship, D=kTμ/q rewrite the flow equation then derive the Nernst equation at equilibrium. (Hint: remember at equilibrium there is a balance and you usually set a the sum of components to zero). $Z = 1$

$$J_{TOTAL} = 0 \qquad Z = 1$$

$$-kT\cancel{\mu}/q\,\frac{d[K]}{dx} - \cancel{\mu}\,Z[K^+]\frac{dV}{dx}$$

$$dV = q\,kT\left(\frac{d[K^+]}{[K^+]}\right)$$

$$\int_{V_1}^{V_2} dV = kTq\int\frac{dk^+}{[K^+]} \qquad V_2 - V_1 = kTq\,\ln\left(\frac{K_1^+}{K_2^+}\right)$$