→ 10 questions
4 pts/question

Short answer. 40 pts.

1. Why are deviation variables used in process control models?

To linearize model and to allow for zero initial conditions.

2. Is the A matrix in a state space model a function of the state variables? Why or why not?

No. Because the model eqns must be linear, and this represents a non-linearity. $\underline{A} = f(\underline{x})$

3. For the ODE (dx/dt = ax + b; b constant), how is the integrating factor used to solve the equation?

$$\frac{d}{dt}\left(e^{-at}x\right) = -ae^{-at}x + e^{-at}\frac{dx}{dt} = be^{-at}$$

integrate: $e^{-at}x\Big|_0^t = -\frac{b}{a}e^{-at}\Big|_0^t$ or $x(t)e^{-at} - x(0) = -\frac{b}{a}\left(e^{-at} - 1\right)$

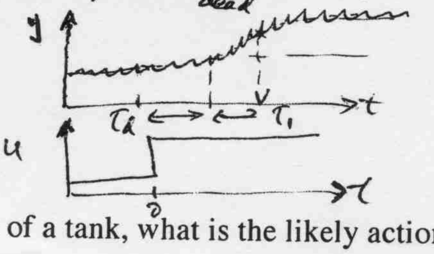
$$\underline{x(t) = x(0)e^{at} - \frac{b}{a}\left(1 - e^{at}\right)} \quad (a < 0 \text{ for } x \to 0 \text{ at } t \to \infty)$$

4. What is meant by 'open loop stability' and how is it different from 'closed loop stability?

- open loop stability means the output is bounded for some bounded input change with <u>no</u> feedback
- closed loop stability means output is bounded for a bounded input <u>with</u> feedback.

5. Describe ~~two~~ one way~~s~~ to fit measured plant data to a first order plus dead time model.

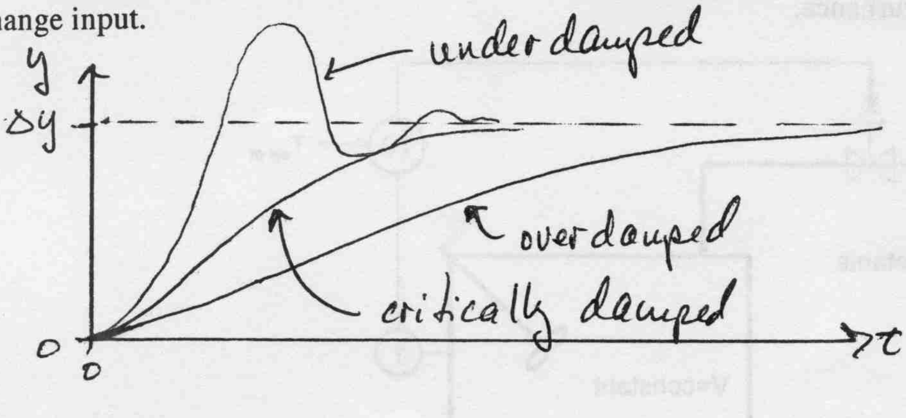
observe approximate dead time τ_{dead} and estimate 63.2% of output change to get τ_{1st}



6. If a control valve is on the outlet of a tank, what is the likely action of the valve (air-to-open or air-to-close)? Why?

for outlet flow, generally want air-to-close, or fail open, so that if air is lost for valve, tank does not overflow.

7. Sketch and identify the three output responses for a second order system to a step change input.



8. What is 'proportional offset' and when does it occur?

A difference between the final process output and the desired set point, due to the use of a P-only controller in the feedback loop. It occurs when the controller gain $K_c < \infty$.

9. What are three consequences of increasing the proportional gain in a feedback control loop?

1. faster response
2. less offset
3. possibility of causing manipulated variable to saturate or may cause closed loop instability.

10. Give one example of how 'dead time' might arise in a process control problem.

- delay in measuring some controlled variable
- higher order systems are sometimes modelled as first order plus dead time.

(a)

$$\cancel{V\varrho c_p}\frac{dT}{dt} = \frac{F(T_0 - T)\cdot \varrho c_p}{V\varrho c_p} + \frac{Q}{V\varrho c_p}$$

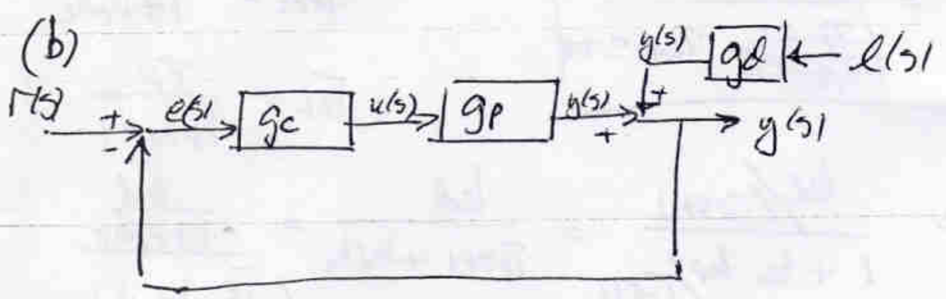
$$\frac{dT}{dt} = \frac{F}{V}(T_0 - T) + \frac{Q}{V\varrho c_p} = \frac{F}{V}(T_0 - T) + Q'$$

$$Q' = \frac{Q}{V\varrho c_p} \; [=] \; K/s$$

at ss $\quad \frac{F}{V} T_s = \frac{F}{V} T_0 + \frac{Q_s}{V\varrho c_p}$

(or) $\quad T_s = T_0 + \frac{Q_s}{F\varrho c_p} \leftarrow$

(b)



y : T
u : F
l : Q

(c) transfer fcns:

· process : $\frac{y(s)}{u(s)} \rightarrow T, F$

$$\frac{dT}{dt} = \frac{F}{V}(T_0 - T) + \frac{Q}{V\varrho c_p}$$

$$\frac{d\tilde{T}}{dt} = \underbrace{-\left(\frac{F_s}{V}\right)}_{-a = -\frac{1}{\tau}}\tilde{T} + \underbrace{\left(\frac{T_0 - T_s}{V}\right)}_{\underbrace{-\left(\frac{T_s - T_0}{V}\right)}_{-b}}\hat{F} + \underbrace{\left(\frac{1}{V\varrho c_p}\right)}_{c}\hat{Q}$$

$$\frac{d\tilde{T}}{dt} = -a\tilde{T} - b\hat{F} + c\hat{Q}$$

LT: $\quad s\,y(s) + a\,y(s) = -b\,u(s) + c\,l(s)$

$$y(s) = \frac{-b\,u(s)}{s+a} + \frac{c\,l(s)}{s+a} = \frac{-(b/a)\,u(s)}{\tau s+1} + \frac{(c/a)\,l(s)}{\tau s+1}$$

$$g_p = \frac{y(s)}{u(s)} = \frac{-(b/a)}{\tau s+1} = \frac{k_p}{\tau_p s+1} \qquad g_d = \frac{(c/a)}{\tau s+1} = \frac{y(s)}{l(s)} = \frac{k_d}{\tau_p s+1}$$

d. $g_c(s) = k_c$

$$g_{CL}^{sp} = \frac{k_c g_p}{1+k_c g_p} = \frac{k_c k_p/(\tau_p s+1)}{1+k_c k_p/(\tau_p s+1)}$$

$$= \frac{k_c k_p}{\tau_p s+1+k_c k_p} = \frac{k_c k_p/(1+k_c k_p)}{\left(\frac{\tau_p}{k_p k_c+1}\right)s+1}$$

$$\boxed{g_{CL}^{sp} = \frac{k_{CL}^{sp}}{\cancel{\left(\frac{\tau_p}{k_p k_c+1}\right)s}\ \tau_{CL} s+1}} \qquad k_{CL}^{sp} = \frac{k_p k_c}{1+k_p k_c} \qquad \tau_{CL} = \frac{\tau_p}{k_p k_c+1}$$

and

$$g_{CL}^{l} = \frac{g_d}{1+k_c g_p} = \frac{k_d/(\tau_p s+1)}{1+k_c k_p/(\tau_p s+1)} = \frac{k_d}{\tau_p s+1+k_d k_c} = \frac{\frac{k_d}{1+k_p k_c}}{\left(\frac{\tau_p}{1+k_p k_c}\right)s+1}$$

$$\boxed{g_{CL}^{l} = \frac{k_{CL}^{l}}{\tau_{CL}^{l} s+1}} \qquad k_{CL}^{l} = \frac{k_d}{1+k_p k_c} \qquad \tau_{CL}^{l} = \tau_{CL}^{p}$$

e. for $r(s) = \frac{\Delta r}{s}$ $\qquad y(s) = r(s)\cdot g_{CL}^{sp}(s) = \frac{\Delta r}{s}\cdot\frac{k_{CL}^{sp}}{\tau_{CL} s+1}$

$$y(t\to\infty) = \lim_{s\to 0} \cancel{s}\cdot\frac{\Delta r}{\cancel{s}}\,\frac{k_{CL}^{sp}}{\tau_{CL} s+1} = \frac{\Delta r\cdot k_p k_c}{1+k_p k_c}$$

double check this answer.

or

$$\boxed{\frac{y}{\Delta r}(t\to\infty) = \frac{k_p k_c}{1+k_p k_c}}$$

$$e = (y-\Delta r) = \left(\Delta r\left(\frac{y}{\Delta r}-1\right)\right) = \Delta r\left(\frac{\cancel{k_p k_c}}{1+k_p k_c} - \frac{(1+\cancel{k_p k_c})}{1+k_p k_c}\right) = \frac{\Delta r}{1+k_p k_c}$$

$$\text{offset} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,e(s)$$

$$= \lim_{s\to 0} s \cdot \left(\frac{\Delta r}{s} - \frac{\Delta r}{s}\,\frac{k_{CL}}{\tau_{CL}s+1} \right)$$

$$= \lim_{s\to 0} \Delta r \left(1 - \frac{k_{CL}}{\tau_{CL}s+1} \right)$$

$$= \Delta r \left(1 - \frac{k_c k_p}{1+k_c k_p} \right)$$

$$= \Delta r\, \frac{1+k_c k_p - k_c k_p}{1+k_c k_p}$$

$$\text{offset} = \frac{\Delta r}{1+k_c k_p}$$