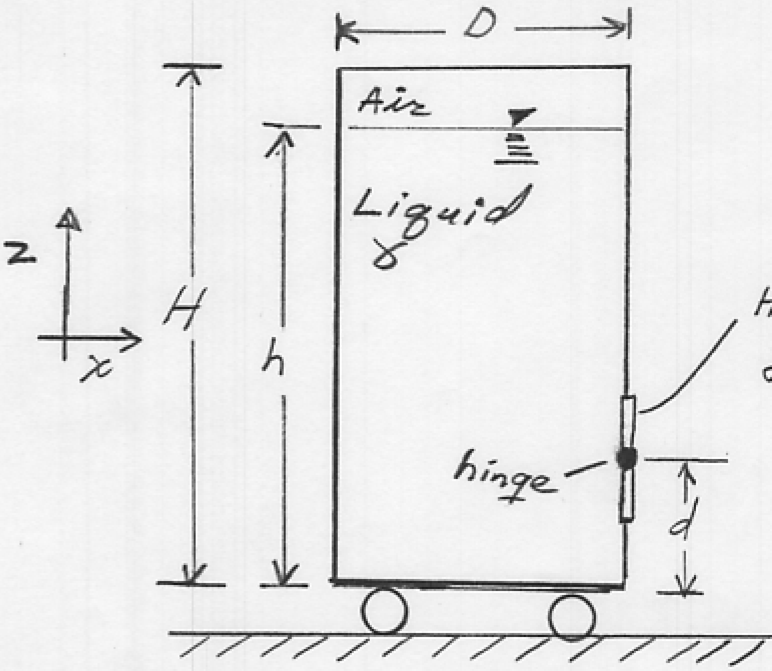
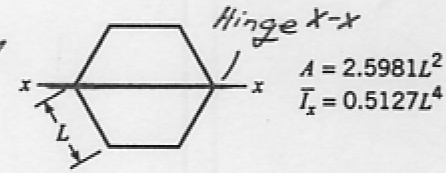
PROBLEM 1: THE COOLER





- diameter of the cooler is D = 1m
- total height H of the cooler is H = 2m.
- hexagonal gate has sides with L=5cm
- center of the hexagonal gate is located d=20cm from the bottom of the cooler.

Figure 1: Wheeled cooler of liquid

A wheeled cooler, filled with an unknown liquid, sits at rest on the the floor. A hexagonal gate, hinged through its center, prevents the liquid from leaving the cooler.

Problem 1a.

Initially, the air in the cooler is open to the atmosphere, and the height h of the fluid in the cooler is h=1.9m. If a torque T=0.03 *N-m* is applied to the hexagonal gate's hinge to keep it from opening, what value of specific weight (γ) will cause the gate to open?

① $F_o = \bar{p} A = \gamma \bar{y} A$ Resultant force on gate

② Moment about hinge $M_o = F_o (y_{cp} - \bar{y})$

$$= \frac{F_o \bar{I}}{\bar{y} A}$$

$$= \frac{(\gamma \bar{y} A) \bar{I}}{\bar{y} A} = \gamma \bar{I}$$

③ $\Sigma M_{Hinge} = T - M_o = 0$

$$\longrightarrow T = M_o = \gamma \bar{I} \longrightarrow \gamma = \frac{T}{\bar{I}} = \frac{0.03}{0.5127(0.05)^4} \frac{N}{m^3}$$

$$= 9362 \frac{N}{m^3}$$

Problem 1b.

At some later time, the air in the cooler is pressurized to p=150kPa absolute. If the gate is removed (more likely blown out!), what force (magnitude and direction) will initially accelerate the cooler due to the exiting liquid jet? Neglect viscous and exit losses, and assume a uniform velocity distribution for the jet. Take the fluid specific weight to be equal to γ = 10kN/m3. $h=1.9m$

① Bernoulli to find exit velocity of jet:

$$\frac{(P_{air} - P_{atm})}{\gamma} + (h-d) = \frac{V_{out}^2}{2g}$$

absolute pressure!

$$V = \left[2g\left(\frac{P_{air}-P_{atm}}{\gamma} + h - d\right)\right]^{1/2}$$

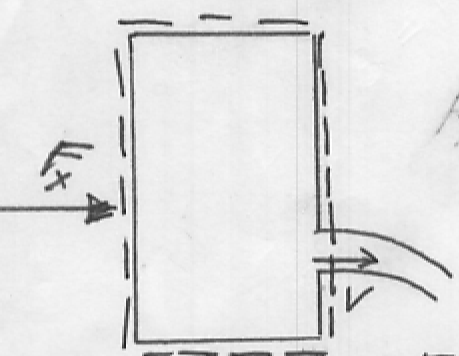
$$= \left[2 \cdot 9.8\left(\frac{150-101.3}{10} + 1.9 - 0.2\right)\right]^{1/2} = 11.35\ \frac{m}{s}$$

② $\sum \vec{F} = \dot{m}_{out} \vec{V}_{out}$

$\vec{V}_{out} = 11.35 \uparrow \frac{m}{s}$; $\dot{m}_{out} = \rho V_{out} A_{out} = \frac{\gamma}{g} V_{out} (2.598 L^2)$ ← From DIAGRAM

$$= \frac{10000 \cdot 11.35 \cdot 2.598 (0.05)^2}{9.81}$$

$$= 75.15\ \frac{kg}{s}$$



$\sum \vec{F} = F_x \uparrow$

$\sum \vec{F} = \dot{m}_{out} \vec{V}_{out}$

$F_x = 75.15 \cdot 11.35 N = 853 N$

FORCE ON COOLER IS EQUAL & OPPOSITE:

$\underline{\underline{853N \text{ to LEFT}}}$ (recoil)

Problem 1c.

For the same conditions described in Problem 1b, at what rate will the free surface in the cooler initially descend?

$$\dot{m}_{out} = \frac{d}{dt}\int_{CV} \rho\, d\forall$$

$$= \rho A_{tank} \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{\dot{m}_{out}}{\rho A_{tank}} = \frac{75.15}{\left(\frac{10{,}000}{9.81}\right)\frac{\pi \cdot 1^2}{4}}\ \frac{m}{s} = \underline{\underline{9.4 \frac{cm}{s}}}$$

PROBLEM 2: THE FALLING PISTON

3.2

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UNIVERSITY OF CALIFORNIA
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College of Engineering

E 103

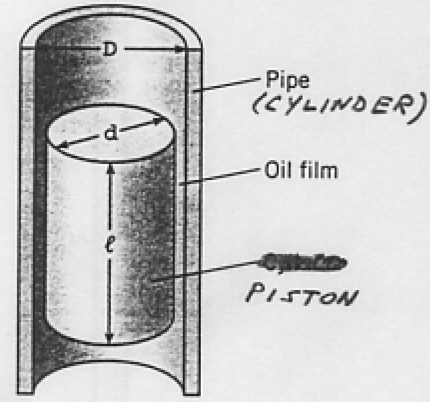
Quiz 1

Open Book

$\ell = 1 ft$ $\Delta y = \frac{0.01}{12} ft$

$d = 9.98/12 ft$

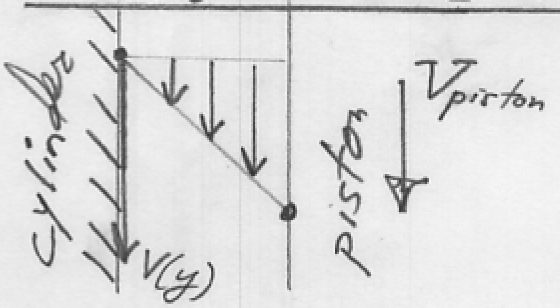
$D = 10/12 ft$



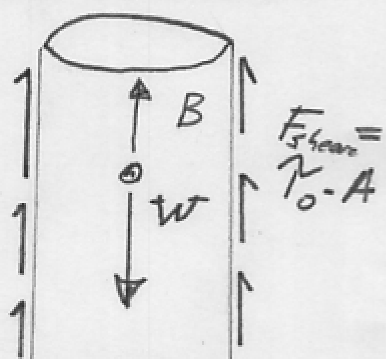
1. A smooth piston 12 in. long and 9.98 in. diameter, which weighs 5 lbs., slides under its own weight inside a 10.00 in diameter cylinder. A continuous film of oil (μ = 10^{-3} lb.sec/ft.^{2}, s = 0.9) is maintained between the piston and cylinders. Calculate the steady rate of fall.

Assume a linear velocity distribution in the oil between the piston and the cylinder.

VELOCITY PROFILE: OIL



FBD: PISTON



In steady fall, the weight balances the shear force and the buoyancy force.

IN FREEFALL, $F_{shear} + B = W$

$$\tau = \mu \frac{dV}{dy}$$

Since V(y) is linear, $\frac{dV}{dy}$ is constant: $\frac{dV}{dy} = \frac{V_{piston} - V_{CYL}}{\Delta y}$

$$= \frac{V_{piston} - 0}{\Delta y}$$

Δy = spacing

The shear stress of the fluid on the piston is given by $\tau_o = \mu \left.\frac{dV}{dy}\right|_{piston} = \frac{\mu V_{piston}}{\Delta y}$; $F_{shear} = \tau_o \cdot A = \frac{\mu V_{piston}}{\Delta y}(\pi d \ell)$

WEIGHT $W = 5 lbf$; Buoyant Force $B = 0.9(62.4 \frac{lbf}{ft^3}) \frac{\pi d^2}{4} \cdot \ell = 30.5 lbf$

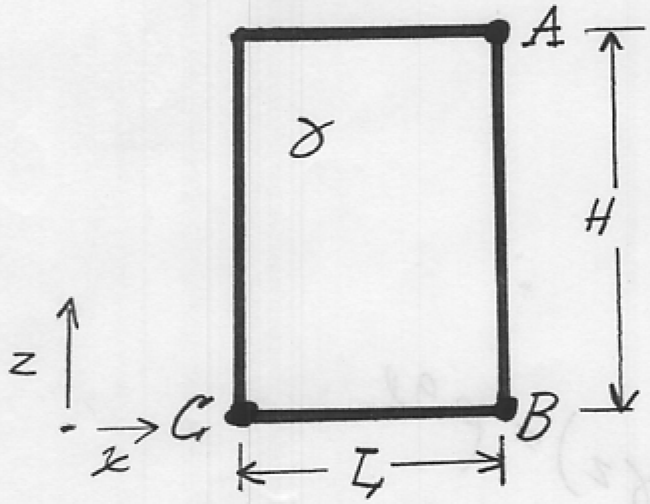
$F_{shear} = W - B$ $\frac{\mu V_{piston} \pi d \ell}{\Delta y} = 5 - 30.5$

$W < B \rightarrow$ piston rises

$V_{piston} = -8.14 \frac{ft}{s}$ (UPWARD!)

PROBLEM 3: ACCELERATING TANK

The closed tank shown is full of liquid having γ = 10 kN/m3. The whole tank is accelerated downward at 2g/3 and accelerated to the right at g . If L = 2m and H = 3m, what is the difference in pressure between points C and A, Pc - Pa?



Euler equation: $-\frac{\partial}{\partial l}(p+\gamma z) = \rho a_l$

x-direction: $-\frac{\partial}{\partial x}(p+\gamma z) = \rho a_x = \rho g$ $\quad a_x = g$

x-integrate from pt. C to pt. B:

$$-\int_C^B \frac{\partial}{\partial x}(p+\gamma z)\,dx = \int_C^B \rho g \, dx$$

$$-(p+\gamma z)\Big|_C^B = \rho g x\Big|_C^B$$

$$-[p_B + \gamma z_B - (p_C + \gamma z_C)] = \rho g (x_B - x_C) = \rho g L$$

$z_B = z_C$, so $p_C - p_B = \rho g L$

z-direction $-\frac{\partial}{\partial z}(p+\gamma z) = \rho a_z = -\rho \frac{2}{3} g$

z-integrate from $B \to A$: $-(p+\gamma z)\Big|_B^A = -\frac{2}{3}\rho g z\Big|_B^A$

$$-[(p_A + \gamma z_A) - (p_B + \gamma z_B)] = -\frac{2}{3}\rho g (z_A - z_B) = -\frac{2}{3}\gamma H$$

$$p_B - p_A = -\frac{2}{3}\gamma H + \gamma H = \frac{\gamma H}{3}$$

$$p_C - p_A = (p_B - p_A) + (p_C - p_B) = \frac{\gamma H}{3} + \gamma L = 10000\,Pa + 20000\,Pa = 30\,kPa$$