## CE 152: CEE Systems Analysis <br> Fall 2003 Final Exam (30 points) <br> Professor Samer Madanat

## Problem 1 (6 points)

Assume that the number of highway miles that can be resurfaced, H , is a function of both the hours of labor, L , and of machines, M :

$$
\mathrm{H}=0.5 \mathrm{~L}^{0.2} \mathrm{M}^{0.8}
$$

(a) Minimize the cost of resurfacing 20 miles, given that the hourly rates for labor and machines are: $C_{L}=\$ 20 ; C_{M}=\$ 160$.
(b) Interpret the meaning of the Lagrange multiplier in this problem.

## Problem 2 (9 points)

City planners must decide the optimal allocation of fire stations to three districts. Zero to three stations may be located in a district. The table below shows the relationship between the number of stations and the annual expected property damage due to fires, based on statistical data. Differences among districts are due to population, construction materials, and so on. A budget constraint restricts the total number of stations to five.

|  | Number of stations per district |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| District | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| 1 | 2.0 | 0.9 | 0.3 | 0.2 |
| 2 | 0.5 | 0.3 | 0.2 | 0.1 |
| 3 | 1.5 | 1.0 | 0.7 | 0.3 |

(a) Determine the optimal allocation by dynamic programming
(b) Could linear programming solve the problem? Why or why not?
(c) Write the dynamic programming recursion.

## Problem 3 (9 points)

Consider the network shown below:


The link performance functions are (in minutes, as functions of flows $x$, in veh./hr):
$\mathrm{t}_{\mathrm{ps}}=10+10 \mathrm{x}_{\mathrm{ps}}$
$\mathrm{t}_{\mathrm{qs}}=60+\mathrm{x}_{\mathrm{qs}}$
$\mathrm{t}_{\mathrm{rp}}=60+\mathrm{x}_{\mathrm{rp}}$
$\mathrm{t}_{\mathrm{rq}}=10+10 \mathrm{x}_{\mathrm{rq}}$
(a) There are 6 vehicles / hour which want to travel from node $r$ to node s. Find the equilibrium flow pattern. Show that this flow pattern is user optimal by transferring one unit of flow from one route to the other.
(b) A highway improvement program adds a directed link from q to p to the network, with link performance function: $\mathrm{t}_{\mathrm{qp}}=10+\mathrm{x}_{\mathrm{qp}}$. Find the new equilibrium flow pattern from $r$ to $s$ and describe what the changes are with respect to part (a).
(c) Compute the total system travel time in cases a) and b). How has it changed? Does this seem intuitively correct? Can you explain why this happened?

## Problem 4 (6 points)

The Mountain Movers Truck Company has $\$ 1,200,000$ to replace its fleet. The company is considering three different truck types:

| Truck Type: | A | B | C |
| :--- | :--- | :--- | :--- |
| Carrying Capacity (tons): | 10 | 20 | 18 |
| Average Speed (mph): | 40 | 35 | 35 |
| Price (in \$1,000): | 48 | 78 | 90 |
| Drivers required: | 1 | 2 | 2 |
| Shifts per day: | 3 | 3 | 3 |
| Hours per day: | 18 | 18 | 21 |

Type C is the improved model of Type B. It has sleeping space for one driver. This reduces the carrying capacity and increases the price. The company employs 150 drivers every day, has 50 overnight parking spaces and wishes to maximize the delivery capacity of its fleet (in ton-mile/day).
(a) Formulate the selection of the trucks as an optimization problem. Explain the decision variables, objective function and constraints.
(b) What assumptions, if any, did you make in this problem?

