Final exam – Fall 2012
Phys. 7B – C. Bordel

1

a) $C_V = \frac{d}{2} k_B$ /at. $= \frac{d}{2} R$ /molecule

$\Longrightarrow$ 5 deg. of freedom $\Longrightarrow$ 2 at./molecule (at moderate T)

$PV = nRT$ then $\boxed{V_1 = \frac{nRT_1}{P_1} = \frac{2RT_1}{P_1}}$

b) $T_f = c^{st} = T_1 \Rightarrow \Delta E_{int} = 0 \Longrightarrow Q = W$

c) $V = c^{st} \Longrightarrow V\,dP = nR\,dT \quad (P\,dV = 0)$

$\boxed{\Delta E_{int} = n\frac{5}{2} R\,\Delta T = Q}$ because $\boxed{W = 0}$

then $\Delta T = \frac{Q}{5R}$ and $\boxed{T_f = T_1 + \frac{Q}{5R}}$

d) $\Delta E_{int} = Q - W$ where $Q = nC_p \Delta T = \frac{7}{2} nR\,\Delta T$

so $\Delta T = \frac{Q}{7R}$ and $\boxed{T_f = T_1 + \frac{Q}{7R}}$

$\Delta E_{int} = nC_V \Delta T = 2 \cdot \frac{5R}{2} \cdot \frac{Q}{7R} = \boxed{\frac{5Q}{7}}$

$P = c^{st} \Longrightarrow P\,dV = nR\,dT$

$\Longrightarrow \boxed{W = nR\,\Delta T = \frac{2Q}{7}}$

2

a) $V = \frac{e}{4\pi\varepsilon_0 r}\exp\left(-\frac{r}{a}\right)$ no angular dependence

$\Rightarrow \vec{E}$ is radial

$$E_r = \frac{-\partial V}{\partial r} = -\frac{-e}{4\pi\varepsilon_0 r^2}\exp\left(-\frac{r}{a}\right) - \frac{e}{4\pi\varepsilon_0 r}\left(-\frac{1}{a}\right)\exp\left(-\frac{r}{a}\right)$$

$$\boxed{\vec{E}(r) = +\frac{e}{4\pi\varepsilon_0}\frac{1}{r^2}\left(1+\frac{r}{a}\right)\exp\left(-\frac{r}{a}\right)\hat{r}}$$

b) $$\Phi_E = \oint_{(A)} \vec{E}.\vec{dA} = 4\pi r^2 E(r) = \boxed{\frac{e}{\varepsilon_0}\exp\left(-\frac{r}{a}\right)\left(1+\frac{r}{a}\right)}$$

c) $$\phi_E = \frac{Q_{encl.}}{\varepsilon_0} \Rightarrow \boxed{Q(r) = e \cdot \exp\left(-\frac{r}{a}\right)\left(1+\frac{r}{a}\right)}$$

$\lim_{r\to+\infty} Q(r) = 0$ bec. atom is neutral

d) $dQ_i = \rho(r)\,dV = \rho(r)\,4\pi r^2 dr \Rightarrow \rho(r) = \frac{1}{4\pi r^2}\frac{dQ}{dr}$

then $$\rho(r) = \frac{e}{4\pi r^2}\left[\frac{1}{a}\exp\left(-\frac{r}{a}\right) + \left(1+\frac{r}{a}\right)\left(-\frac{1}{a}\right)\exp\left(-\frac{r}{a}\right)\right]$$

$$= \boxed{-\frac{e}{4\pi a^2 r}\exp\left(-\frac{r}{a}\right)}$$

3 a) $R = \frac{\rho \ell}{A}$, $\Delta \ell = \alpha \ell \Delta T$, $\Delta A = 2\alpha A \Delta T$ so $R_2 = \frac{\rho_2 \ell_2}{A_2} = \rho_2 \frac{\ell_1 (1+\alpha \Delta T)}{A_1(1+2\alpha\Delta T)}$

and we get $\boxed{\rho_2 = \frac{(1+2\alpha\Delta T)}{(1+\alpha\Delta T)} \cdot \frac{A_1 R_2}{\ell_1}}$

we expect $\rho_2 > \rho_1$ because resistivity is due to e^-/atom collisions and they occur more frequently at higher temperature; atoms move more about their equilibrium position which increases the collision rate in the metal.

b) $\ell_2 - \ell_1 = \alpha \ell_1 (T_2 - T_1) \Longleftrightarrow \boxed{T_2 = \frac{\ell_2 - \ell_1}{\alpha \ell_1} + T_1}$

c) $P = \sigma T_2^4 (2\cancel{A_2} + 2\pi r_2 \ell_2)$ $\qquad A_2 \ll r_2 \ell_2$

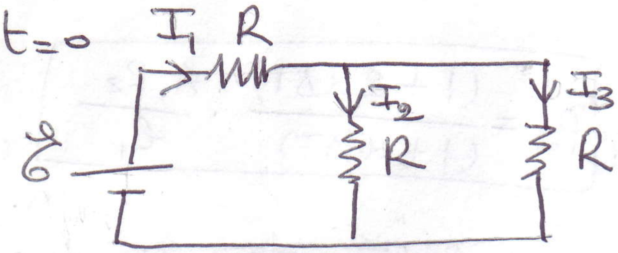
$= \sigma T_2^4 \left(2\pi \sqrt{\frac{A_2}{\pi}} \ell_2\right)$

$= 2\sqrt{\pi}\sqrt{A_2}\,\ell_2 \sigma T_2^4$

d) $\boxed{P_{net} = \sigma (T_2^4 - T_1^4)(2\sqrt{\pi A_2}\,\ell_2)}$

4

a) $t=0$

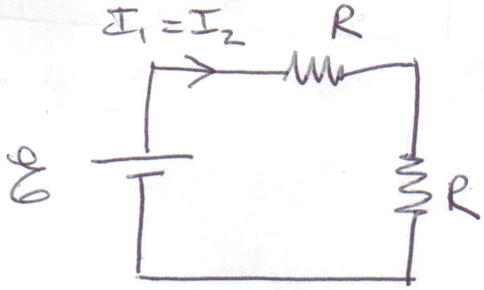


or

$I_2 = I_3 = \frac{1}{2} I_1$ then $\boxed{I_1 = \frac{\mathcal{E}}{\frac{3R}{2}} = \frac{2\mathcal{E}}{3R}}$

the capacitor behaves like an ideal wire at $t \sim 0$. So $\boxed{I_2 = I_3 = \frac{\mathcal{E}}{3R}}$

b) $t = \infty$ → capa. behaves like an open circuit



then $\boxed{\begin{array}{l} I_3 = 0 \\ I_1 = I_2 = \frac{\mathcal{E}}{2R} \end{array}}$ and

c) $\begin{cases} RI_1 + RI_3 + \frac{Q}{C} - \mathcal{E} = 0 \\ RI_1 + RI_2 - \mathcal{E} = 0 \\ I_1 = I_2 + I_3 \end{cases}$

(1) gives $R\frac{dI_1}{dt} + R\frac{dI_3}{dt} + \frac{1}{C}\frac{dQ}{dt} = 0$

(2) and (3) combined give: $RI_1 + R(I_1 - I_3) - \mathcal{E} = 0$

$\Leftrightarrow 2RI_1 - RI_3 - \mathcal{E} = 0$

then $2R\frac{dI_1}{dt} - R\frac{dI_3}{dt} = 0$

which by substitution into (1) gives:

$R\frac{dI_3}{dt} + R\frac{dI_3}{dt} + \frac{I_3}{C} = 0$

then $\boxed{I_3(t) = \frac{\mathcal{E}}{3R}\exp\left(-\frac{2t}{3RC}\right)}$

therefore $I_1(t) = \frac{\mathcal{E} + RI_3}{2R} = \frac{\mathcal{E}}{2R} + \frac{\mathcal{E}}{6R}\exp\left(-\frac{2t}{3RC}\right)$

and $I_2(t) = I_1 - I_3 = \frac{\mathcal{E}}{2R} + \frac{\mathcal{E}}{6R}\exp\left(\frac{-2t}{3RC}\right) - \frac{\mathcal{E}}{3R}\exp\left(\frac{-2t}{3RC}\right)$

$= \frac{\mathcal{E}}{2R} - \frac{\mathcal{E}}{6R}\exp\left(-\frac{2t}{3RC}\right)$

d) Q = constant because battery is removed

$\Rightarrow V_c = \frac{Q_{max}}{KC}$

where $Q_{max} = C(\mathcal{E} - RI_1(\infty) - RI_3(\infty))$

$= C\left(\mathcal{E} - R \cdot \frac{\mathcal{E}}{2R} - R \cdot 0\right)$

$= \frac{\mathcal{E}C}{2}$

so $V_c = \frac{\mathcal{E}\cancel{C}}{2K\cancel{C}} = \boxed{\frac{\mathcal{E}}{2K}}$

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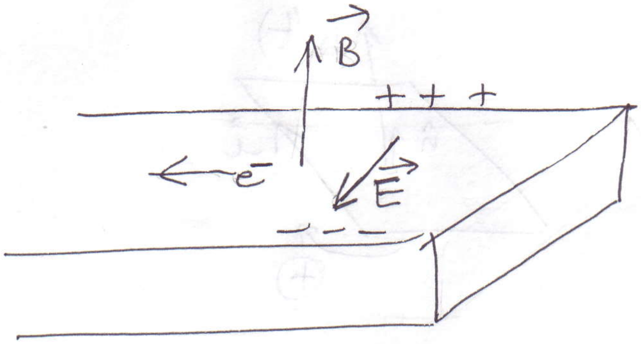
a) n = free e^- density



Lorentz force on e^-:

$\vec{F}_L = -e\,\vec{v}_d \times \vec{B} \Rightarrow F_L = e v_d B$

$\Rightarrow$ electric field points from back to front

$$I = \iint \vec{j}\cdot d\vec{A} = + n e v_d w t \iff v_d = \frac{I}{newt}$$

When equilibrium is reached: $eE_H = evB$

then $$E_H = \frac{IB}{newt}$$

and $$\boxed{\mathcal{E}_H = E_H \cdot w = \frac{IB}{net}}$$

b)
$$\boxed{v_d = \frac{I}{newt} = \frac{\mathcal{E}_H}{B \cdot w}}$$

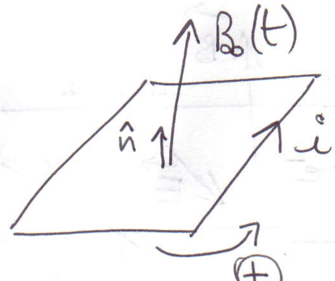
c) $$\boxed{n = \frac{IB}{et\,\mathcal{E}_H}}$$

d) $n = \dfrac{d_{Cu}}{M_{Cu}} \times 1$ then $$\boxed{E_H = \frac{IBM_{Cu}}{d_{Cu}\, etW}}$$

(6) a) $\mathcal{E} = -\dfrac{d\phi_B}{dt}$ where $\dfrac{d\phi_B}{dt} = \dfrac{d}{dt}\left(\iint \vec{B}\cdot d\vec{A}\right)$

$$\frac{d\phi_B}{dt} = \frac{d}{dt}\left(4d^2\,(1-kt)\right) = -4kd^2$$

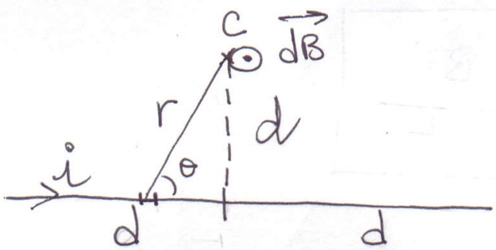
then $\boxed{\mathcal{E} = 4kd^2}$



b) $i = \dfrac{\mathcal{E}}{R} = \dfrac{4kd^2}{R} > 0$

Induced current is counter clockwise so that induced $\vec{B}$ field opposes weakening $\vec{B}_0$ at the origin of negative $\dfrac{d\phi_B}{dt}$.

c) Biot-Savart: $d\vec{B} = \dfrac{\mu_0 I\, d\vec{l} \times \hat{r}}{4\pi r^2}$



all $d\vec{B}$ contributions point out of the page (= z axis).

$$dB_z = \frac{\mu_0\, i}{4\pi\,(x^2+d^2)}\,dx\cdot\sin\theta$$

$$= \frac{\mu_0\, i\, d\, dx}{4\pi\,(x^2+d^2)^{3/2}}$$

then $B_{side} = \dfrac{\mu_0\, i\, d}{4\pi}\displaystyle\int_{-d}^{d}\frac{dx}{(x^2+d^2)^{3/2}}$

$$= \frac{\mu_0\, i\, d}{4\pi}\left[\frac{x}{d^2\sqrt{x^2+d^2}}\right]_{-d}^{+d} = \boxed{\frac{\mu_0\, i}{2\pi d\sqrt{2}} = \frac{\mu_0\, i\sqrt{2}}{4\pi d}}$$

d) $B_{tot}\,(\text{center}) = 4\,B_{side} = \boxed{\dfrac{\mu_0\, i\sqrt{2}}{\pi d}}$

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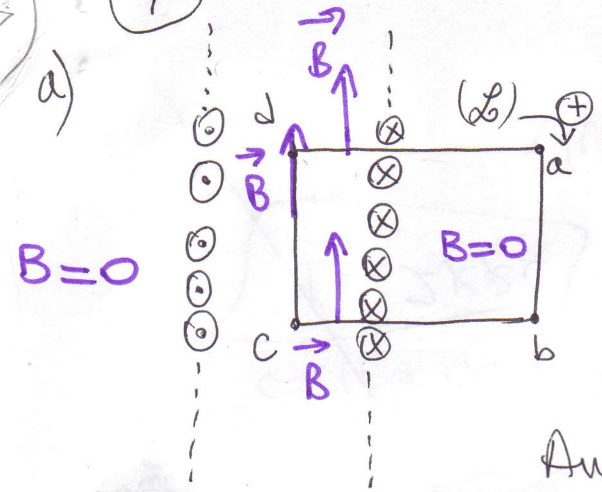
a)



Amperian loop ($\mathcal{L}$)

$$\oint_{(\mathcal{L})} \vec{B}\cdot\vec{dl} = \cancel{\int_a^b \vec{B}\cdot\vec{dl}}_{(B=0)} + \cancel{\int_b^c \vec{B}\cdot\vec{dl}}_{(\perp)} + \int_c^d \vec{B}\cdot\vec{dl} + \cancel{\int_d^a \vec{B}\cdot\vec{dl}}_{(\perp)}$$

$$= B\ell$$

Ampère's law : $\oint_{(\mathcal{L})} \vec{B}\cdot\vec{dl} = \mu_0 I_{net}$

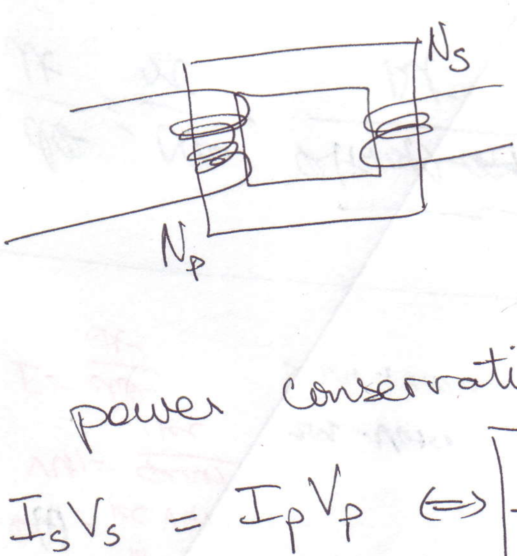
then $B\ell' = \dfrac{\mu_0 N_p I \ell'}{\ell}$

and therefore $\boxed{B = \dfrac{\mu_0 N_p I}{\ell}}$

$$L = \frac{N_p \Phi_B}{I} = \frac{N_p B A}{I} = \frac{\mu_0 N_p^2 A}{\ell}$$

b) $U = \dfrac{1}{2} L I^2 = \dfrac{\mu_0 N_p^2 A}{2\ell} \cdot I^2$

c)



$\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p} \iff \boxed{N_s = N_p \cdot \dfrac{V_s}{V_p}}$

Num : $N_s = 15$

d) power conservation for ideal transformer

$I_s V_s = I_p V_p \iff \boxed{\dfrac{I_s}{I_p} = \dfrac{V_p}{V_s}}$ then $\dfrac{I_s}{I_p} = \dfrac{110}{15}$