MATH 140 - FINAL 12/14/05 D. Geba

1.(2p) Show that a surface of revolution can always be parametrized so that

$$E = E(v), \quad F = 0, \quad G = 1.$$

2.(1p) Let S be a surface of revolution and C its generating curve. Let s be the arc length of C and denote by $\rho = \rho(s)$ the distance to the rotation axis of the point of C corresponding to s. Show that the area of S is

$$2\pi \int_0^l \rho(s)\, ds$$

where l is the length of C.

3.(2p) Show that the meridians of a torus are lines of curvature. (Parametrization: $X(u,v) = ((a + r\cos u)\cos v, (a + r\cos u)\sin v, r\sin u)$)

4.(1p) If the surface S_1 intersects the surface S_2 along the regular curve C, then the curvature k of C at $p \in C$ is given by

$$k^2 \sin^2\theta = \lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2\cos\theta$$

where λ_1 and λ_2 are the normal curvatures at p, along the tangent line to C, of S_1 and S_2, respectively, and θ is the angle made up by the normal vectors of S_1 and S_2 at p.

5.(2p) Prove that the cylinder and the saddle $z = x^2 - y^2$ are not locally isometric.

6.(1p) Consider two meridians of a sphere C_1 and C_2 which make an angle ϕ at the point p. Take the parallel transport of the tangent vectors w_1 of C_1, along C_1, and w_2 of C_2, along C_2, from the initial point p to the point q where the two meridians meet again, obtaining respectively, w_1' and w_2'. Compute the angle between w_1' and w_2'.